Graphical Interpretation of the Replicating Portfolio

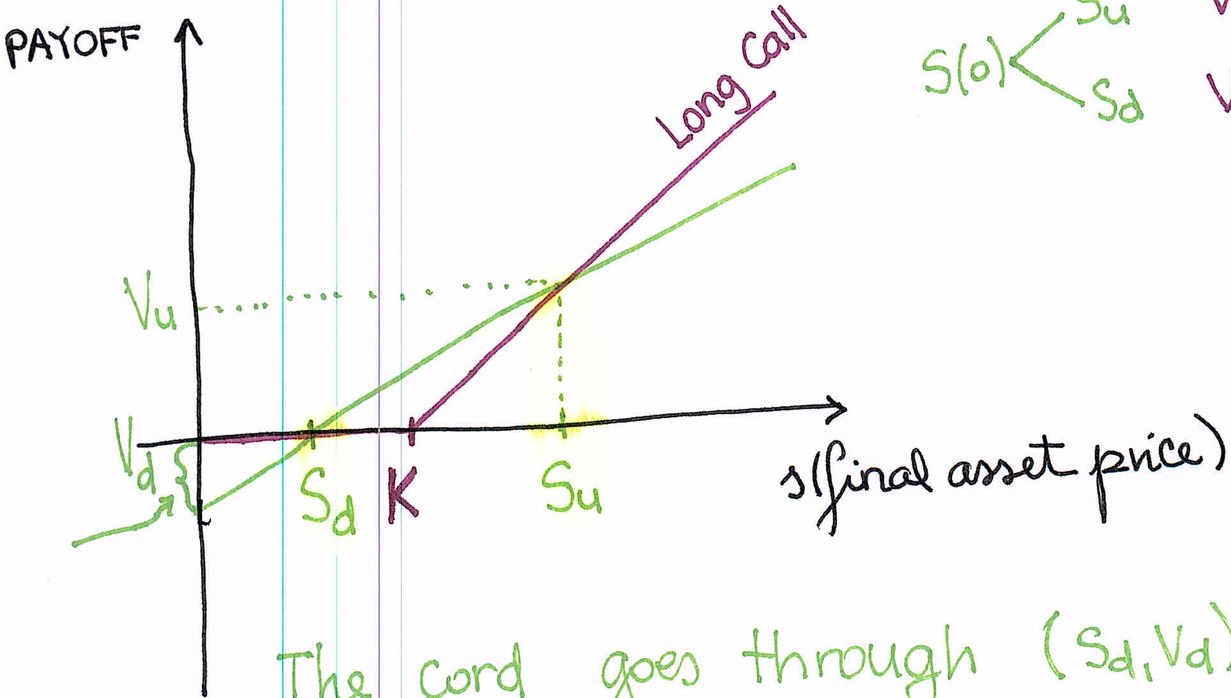
Special case: A CALL OPTION on a NON-DIVIDEND-PAYING asset.

$$S(0) \begin{cases} S_u \\ S_d \end{cases}$$

PAYOFF

$V_u = (S_u - K)_+$

$V_d = (S_d - K)_+$



The cord goes through (S_d, V_d) and (S_u, V_u)

- slope: $\dfrac{V_u - V_d}{S_u - S_d} = \Delta > 0 \Rightarrow$ Buy shares.
- the intercept : $Be^{rh} < 0 \Rightarrow$ Borrowing @ risk-free rate (r).

THE RISK-NEUTRAL PRICING FORMULA

$$V_c(0) = e^{-rT}\left[p^*(S_u - K)_+ + (1-p^*)(S_d - K)_+\right]$$

$$V_p(0) = e^{-rT}\left[p^*(K - S_u)_+ + (1-p^*)(K - S_d)_+\right]$$

$$\text{w/ } p^* = \frac{e^{(r-\delta)h} - d}{u - d}$$

Q: What is $\underbrace{V_c(0) - V_p(0)}_{\text{Put-call Parity : ☺}}$?

3. You are given the following regarding stock of Widget World Wide (WWW):

(i) The stock is currently selling for $50.

(ii) One year from now the stock will sell for either $40 or $55.

$S(0)=50$, $S_u=55$, $S_d=40$

(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 10%. $\delta=0.10$

The continuously compounded risk-free interest rate is 5%. $\boxed{r=0.05}$

While reading the Financial Post, Michael notices that a one-year at-the-money European call written on stock WWW is selling for $1.90. Michael wonders whether this call is fairly priced. He uses the binomial option pricing model to determine if an arbitrage opportunity exists.

$T=1,\ K=50,\ V_c^{obs}(0)=1.90$

What transactions should Michael enter into to exploit the arbitrage opportunity (if one exists)?

(A) No arbitrage opportunity exists.

$\boxed{\text{(B)}}$ Short shares of WWW, lend at the risk-free rate, and buy the call priced at $1.90.

× (C) Buy shares of WWW, borrow at the risk-free rate, and buy the call priced at $1.90. → Pile on risk! :(

(D) Buy shares of WWW, borrow at the risk-free rate, and short the call priced at $1.90. → Pile on risk! :(

× (E) Short shares of WWW, borrow at the risk-free rate, and short the call priced at $1.90.

$V_c(0)$ → repl. portfolio $\Delta=0.3$, $B=-12.68$ → risk-neutral $p^*=0.504$

$V_c(0)=2.4>1.9=V_c^{obs}(0)$ // $\Rightarrow$ $\boxed{\text{B.}}$

$V(0) \; ? \; V^{obs}(0)$

$=$

$\Downarrow$

No arbitrage!

$\neq$

Yes! arbitrage!

$\boxed{V(0) > V^{obs}(0)}$

$\Downarrow$

- Buy the observed one
- short the replicating portfolio

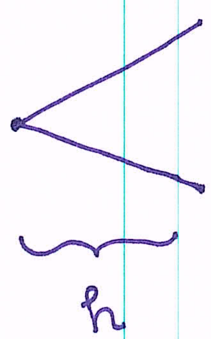
$\boxed{V(0) < V^{obs}(0)}$

- Write the observed
- Long the replicating portfolio

The Forward Binomial Tree

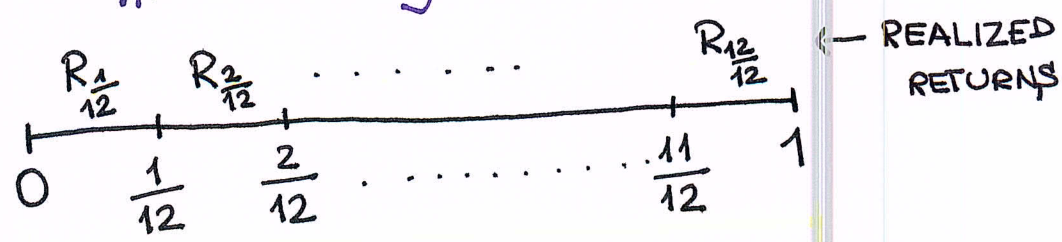
$u, d = ?$

σ.... the standard deviation of the continuously compounded rate of return on the annual time-scale



Toy example. h... one month, i.e. $1/12$

σ_h... monthly volatility



Assume:

- Rates of return are INDEPENDENT and IDENTICALLY DISTRIBUTED over DISJOINT time period of EQUAL LENGTH.

$$\sigma^2 = 12 \cdot \sigma_h^2 \implies \sigma_h = \sigma\sqrt{\tfrac{1}{12}}$$

$$\boxed{\sigma_h = \sigma\sqrt{h}}$$

Recall: $F_{0,h}(S) = S(0)e^{(r-\delta)h}$

In the forward tree:

$$S_u = F_{0,h}(S)\,e^{\sigma\sqrt{h}} = S(0)\,\overbrace{e^{(r-\delta)h+\sigma\sqrt{h}}}^{=:u}$$

$$S_d = F_{0,h}(S)\,e^{-\sigma\sqrt{h}} = S(0)\,\underbrace{e^{(r-\delta)h-\sigma\sqrt{h}}}_{=:d}$$

- $S_u / S_d = e^{2\sigma\sqrt{h}}$
- Risk-neutral probability $p^* = ?$
- Q: Is there a need to check that the NO-ARBITRAGE CONDITION holds?