Early exercise option

\[ t \in [0, T] \]

Admissible exercise dates

the

MAXIMUM

is the

value of

the

option

@ time- \( t \)

\text{vs.}

Continuation value

Employee stock options
Several years ago, John bought three separate 6-month options on the same stock.

- Option I was an American-style put with strike price 20.
- Option II was a Bermuda-style call with strike price 25, where exercise was allowed at any time following an initial 3-month period of call protection.
- Option III was a European-style put with strike price 30.

When the options were bought, the stock price was 20.

When the options expired, the stock price was 26.

The table below gives the maximum and minimum stock price during the 6 month period:

<table>
<thead>
<tr>
<th>Time Period</th>
<th>1st 3 months of Option Term</th>
<th>2nd 3 months of Option Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Stock Price</td>
<td>24</td>
<td>28</td>
</tr>
<tr>
<td>Minimum Stock Price</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

John exercised each option at the optimal time.

Rank the three options, from highest to lowest payoff.

(A)  I > II > III
(B)  I > III > II
(C)  II > I > III
(D)  III > I > II
(E)  III > II > I
Ⅲ. Payoff @ time $-\frac{1}{2}$: $(30-26)_+ = 4$

Ⅰ. The overall minimum stock price is 18:
   The Payoff is $(20-18)_+ = 2$

Ⅱ. The maximal stock price after the lockout period is 28
   The Payoff is $(28-25)_+ = 3$

$\Rightarrow (E)$. 
4. For a stock, you are given:

(i) The current stock price is $50.00.

(ii) $\delta = 0.08$

(iii) The continuously compounded risk-free interest rate is $r = 0.04$.

(iv) The prices for one-year European calls $(C)$ under various strike prices $(K)$ are shown below:

<table>
<thead>
<tr>
<th>$K$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$40$</td>
<td>$9.12$</td>
</tr>
<tr>
<td>$50$</td>
<td>$4.91$</td>
</tr>
<tr>
<td>$60$</td>
<td>$0.71$</td>
</tr>
<tr>
<td>$70$</td>
<td>$0.00$</td>
</tr>
</tbody>
</table>

You own four special put options each with one of the strike prices listed in (iv). Each of these put options can only be exercised immediately or one year from now.

Determine the lowest strike price for which it is optimal to exercise these special put option(s) immediately.

(A) $40$
(B) $50$
(C) $60$
(D) $70$
(E) It is not optimal to exercise any of these put options.
The owner can<br><br>EXERCISE IMMEDIATELY<br><br>Hold on to the<br>EUROPEAN PUT<br><br>Compare: value of immediate exercise to today's put-option price<br><br>Put-call parity:<br<br>$V_p(40) = V_c(40) + 40e^{-0.04} - 50e^{-0.08} = 1.39$
American Options

[Sections 10.4 and 11.1 \rightarrow Sect. 9.3]

Pricing with multiperiod binomial model \rightarrow T: expiration date

- Properties of call and put prices

Early exercise

- Exercise: e.g., if American call, the owner gets \((S(t) - K)^+\) at time-\(t\)

- Properties of option prices for calls and puts: bounds.

- Pricing in a binomial model

- Early exercise: Pros and Cons

Agenda:

- Sections 10.4 and 11.1
American Options (cont'd)

- The valuation of American options proceeds as follows
  - At each node, we check for early exercise
  - If the value of the option is greater when exercised, we assign that value to the node. Otherwise, we assign the value of the option unexercised
  - We work backward through the tree as usual

\[
V_u^A = \begin{cases} \phi^* & \text{if } \phi^* > V_u \\ V_u & \text{otherwise} \end{cases}
\]

\[
V_u = \max(V_u^A, V_u)
\]

\[
V_0^A = \begin{cases} \phi^* & \text{if } \phi^* > V_0 \\ V_0 & \text{otherwise} \end{cases}
\]

\[
V_0 = \max(V_0^A, V_u)
\]
**Exercise Immediately**

\[ IE_u = \hat{v}(S_u) \]

**Decision:**
- **Continue To Hold The Option**
  \[ CV_u = e^{-r_h} \left[ p^* V_{uu} + (1-p^*)V_{ud} \right] \]

\[ \Rightarrow V^A_u = \max \left[ IE_u, CV_u \right] \]
  and the option owner decides/behaves according to which is worth more.

**Down Node**

\[ V^A_d = \max \left[ IE_d, CV_d \right] \]
\[ = \max \left[ \hat{v}(S_d), e^{-r_h} \left[ p^* V_{ud} + (1-p^*)V_{dd} \right] \right] \]

**Root Node**

\[ V^A(0) = \max \left[ IE_0, CV_0 \right] \]
\[ = \max \left[ \hat{v}(S(0)), e^{-r_h} \left[ V^A_u \cdot p^* + V^A_d (1-p^*) \right] \right] \]
4. For a two-period binomial model, you are given:

(i) Each period is one year.
(ii) The current price for a nondividend-paying stock is 20.
(iii) $u = 1.2840$, where $u$ is one plus the rate of capital gain on the stock per period if the stock price goes up.
(iv) $d = 0.8607$, where $d$ is one plus the rate of capital loss on the stock per period if the stock price goes down.
(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of an American call option on the stock with a strike price of 22.

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
Steps:

1st
Populate the stock-price tree using the given values of $u$ and $d$

2nd
Use the pricing procedure to move backwards through the tree and get $V^A(0)$

$n = 2, h = 1 \implies T = n \cdot h = 2 \ldots$ expiration date

$S(0) = 20$

$S_u = u \cdot S(0) \approx 25.68$

$S_d < K$

$S_{dd} < K$

$V_{uu} = 32.97$

$V_{ud} = 22.10$

$V_{dd} = 0.10$

$S_{ud} = 2.48$

$IE_{u} = 3.68$

$IE_{d} = 0$

Risk-neutral probability:

$p^* = \frac{e^{(r-q)h} - d}{u - d} = \frac{e^{0.05 \cdot 0.8607} - 0.8607}{1.2840 - 0.8607} = 0.45$

Complete the sol'n @ home?

Q: Was there early exercise? ←

Q: Why? The "bigger truth"?

Q: How would you proceed in a multiperiod tree?