Currency-option pricing.

NOV 30TH

Analogy:

Foreign currency $\longleftrightarrow$ Continuous-dividend-paying stocks

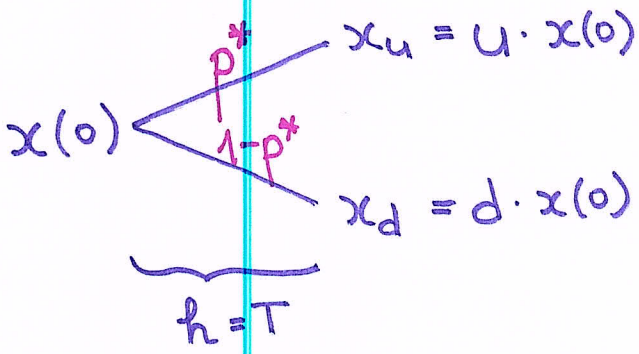
$x(t),\ t \geq 0$ $\longleftrightarrow$ $\underline{\underline{S(t)}},\ t \geq 0$

exchange rate

r_F ... cont.-comp., risk free interest rate for the FC $\longleftrightarrow$ δ ... dividend yield

Q: How do we adapt this analogy for binomial option pricing?

$\boxed{\text{one period}}$



PAYOFF — REPLICATING PORTFOLIO

$V_u = v(x_u) = \Delta e^{r_F T} \cdot x_u + B e^{r_D T}$

$V_d = v(x_d) = \Delta e^{r_F T} \cdot x_d + B e^{r_D T}$

Complete analogy!

Δ ... the number of units of the FC bought @ time-0 deposited to earn risk-free interest @ the rate r_F for the duration of a single period

B ... the risk-free (@ the r_D) investment in the $\underline{\underline{DC}}$

$$p^* = \frac{e^{(r_D - r_F)h} - d}{u - d}$$

$\boxed{\text{THE DEFINITION!}}$

Sample MFE

$ = DC

5. Consider a 9-month dollar-denominated American put option on British pounds.

You are given that: $T = 3/4$ $\qquad$ £ = FC (underlying asset)

(i) The current exchange rate is 1.43 US dollars per pound. $x(0) = 1.43$ ($/£)

(ii) The strike price of the put is 1.56 US dollars per pound. $\boxed{K = 1.56}$

(iii) The volatility of the exchange rate is $\sigma = 0.3$. $\sigma = 0.3$

(iv) The US dollar continuously compounded risk-free interest rate is 8%. $r_D = 0.08$

(v) The British pound continuously compounded risk-free interest rate is 9%. $r_F = 0.09$

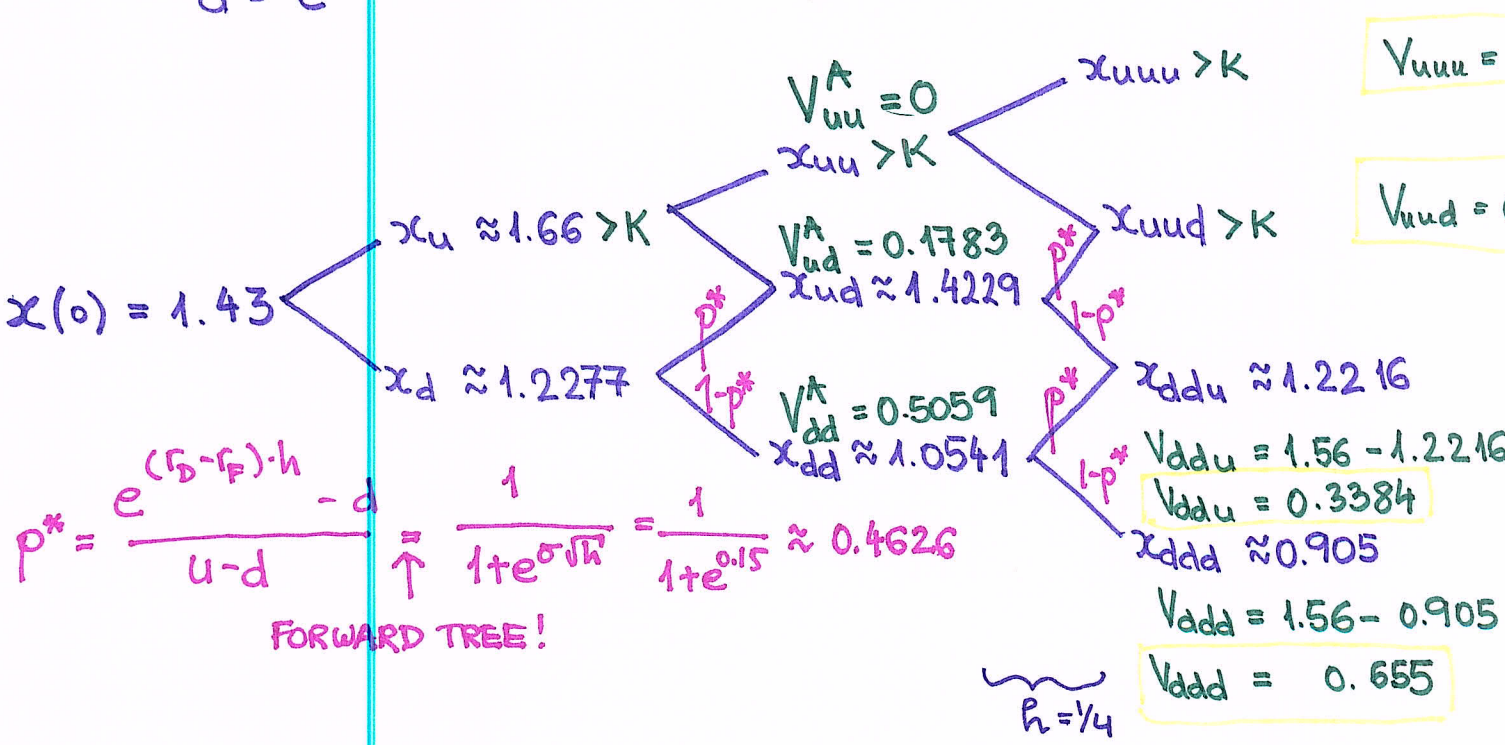
Using a three-period binomial model, calculate the price of the put.

$n = 3 \Rightarrow h = 1/4$ ... length of every period

Q: $u, d = ?$ Forward binomial tree is the "default".

$$u = e^{(r_D - r_F)\cdot h + \sigma\sqrt{h}} = e^{(0.08-0.09)\cdot(1/4) + 0.3(1/2)} = 1.1589$$

$$d = e^{(r_D - r_F)\cdot h - \sigma\sqrt{h}} = e^{-0.01\cdot 0.25 - 0.15} = 0.8586$$



$$p^* = \frac{e^{(r_D - r_F)\cdot h} - d}{u - d} \underset{\uparrow}{=} \frac{1}{1+e^{\sigma\sqrt{h}}} = \frac{1}{1+e^{0.15}} \approx 0.4626$$

FORWARD TREE!

$\boxed{ud}$ · $IE_{ud} = 1.56 - 1.4229 = 0.1371$

· $CV_{ud} = e^{-0.08 \cdot \frac{1}{4}} (1 - 0.4626) \cdot 0.3384 \approx 0.1783$

$\Rightarrow \quad Y^A_{ud} = 0.1783$ (continue!)

$\boxed{dd}$ · $IE_{dd} = 1.56 - 1.0541 \approx \underline{0.5059}$

· $CV_{dd} = e^{-0.02} \left[0.4626 \cdot 0.3384 + 0.5374 \cdot 0.655 \right] \approx 0.4985$

$\Rightarrow \quad Y^A_{dd} = 0.5059$ (Early exercise!)

$\boxed{d}$ · $IE_d = 1.56 - 1.2277 = 0.3323$

· $CV_d = e^{-0.02} \left[0.4626 \cdot 0.1783 + 0.5374 \cdot 0.5059 \right] \approx 0.3474$

$\Rightarrow \quad Y^A_d = 0.3474$ (Continue!)

$\boxed{u}$ ....

$\boxed{\text{ROOT}}$ ....