Currency option pricing.

Analogy:
Foreign currency $\leftrightarrow$ Continuous-dividend-paying stocks

$x(t), t \geq 0$ $\leftrightarrow$ $S(t), t \geq 0$

$\Gamma_F$ ... cont. comp. risk-free interest rate for the FC $\leftrightarrow$ $\delta$ ... dividend yield

Q: How do we adapt this analogy for binomial option pricing?

\[ \begin{align*}
\text{one period} & \\
\text{PAYOFF} & \\
V_{u} &= u(x) = \Delta e^{r_{F}T}x_{u} + B e^{r_{F}T} \\
V_{d} &= d(x) = \Delta e^{r_{F}T}x_{d} + B e^{r_{F}T} \\
\text{Complete analogy?} & \\
\end{align*} \]

$\Delta$ ... the number of units of the FC bought @ time-0 deposited to earn risk-free interest @ the rate $r_{F}$ for the duration of a single period

$B$ ... the risk-free (@ the $r_{F}$) investment in the FC

\[ p^{*} = \frac{e^{(r_{F}-r_{F})h} - d}{u-d} \]

THE DEFINITION?
5. Consider a 9-month dollar-denominated American put option on British pounds.

You are given that: \( T = \frac{3}{4} \)

\( \mathbb{E} = \text{FC (underlying asset)} \)

(i) The current exchange rate is 1.43 US dollars per pound.

(ii) The strike price of the put is 1.56 US dollars per pound. \[ K = 1.56 \]

(iii) The volatility of the exchange rate is \( \sigma = 0.3 \).

(iv) The US dollar continuously compounded risk-free interest rate is 8%.

(v) The British pound continuously compounded risk-free interest rate is 9%.

Using a three-period binomial model, calculate the price of the put.

\( n = 3 \Rightarrow h = \frac{1}{4} \ldots \text{length of every period} \)

\( u, d = ? \)

\( u = e^{(r_f - r_d) \cdot h + \sigma \sqrt{h}} = e^{(0.08 - 0.09) \cdot (\frac{1}{4}) + 0.3 (\frac{1}{2})} = 1.589 \)

\( d = e^{(r_f - r_d) \cdot h - \sigma \sqrt{h}} = e^{(-0.01 \cdot 0.25) = 0.15} = 0.8586 \)

\( V_{uu} = 0 \)

\( V_{uu} > K \)

\( V_{uuu} > K \)

\( x_u \approx 1.66 > K \)

\( x_d \approx 1.2277 \)

\( p^* = \frac{e^{(r_f - r_d) \cdot h} - d}{u - d} = \frac{1}{1 + e^{-0.15}} \approx 0.4626 \)

\( \mathbb{P} = \frac{1}{4} \)

\( V_{dud} = 0.1783 \)

\( V_{udd} > K \)

\( x_{dd} \approx 1.0541 \)

\( V_{ddu} = 0.5059 \)

\( V_{ddd} = 0.3384 \)

\( V_{dud} = 1.56 - 0.2216 \)

\( V_{udd} = 0.3384 \)

\( V_{udd} = 0.905 \)

\( V_{add} = 1.56 - 0.905 \)

\( V_{add} = 0.655 \)

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\[ \begin{align*}
  &\text{IE}_{ud} = 1.56 - 1.4229 = 0.1371 \\
  &\text{CV}_{ud} = e^{-0.08 \times \frac{1}{2}} (1 - 0.4626) \cdot 0.3384 \approx 0.1783 \\
  \Rightarrow &\quad V^A_{ud} = 0.1783 \quad \text{(continue!)}
\end{align*} \]

\[ \begin{align*}
  &\text{IE}_{dd} = 1.56 - 1.0541 = 0.5059 \\
  &\text{CV}_{dd} = e^{-0.02} \left[ 0.4626 \cdot 0.3384 + 0.5374 \cdot 0.655 \right] \approx 0.4985 \\
  \Rightarrow &\quad V^A_{dd} = 0.5059 \quad \text{(Early exercise \%)}
\end{align*} \]

\[ \begin{align*}
  &\text{IE}_d = 1.56 - 1.2277 = 0.3323 \\
  &\text{CV}_d = e^{-0.02} \left[ 0.4626 \cdot 0.1783 + 0.5374 \cdot 0.5059 \right] \approx 0.3474 \\
  \Rightarrow &\quad V^A_d = 0.3474 \quad \text{(Continue \%)}
\end{align*} \]