OPTIONS ON CURRENCY / EXCHANGE RATE

- underlying asset ... "foreign currency" FC
- asset price .... exchange rate from the FC to the "domestic" currency DC

e.g., 0.01 $/¥ (DC / FC)

1 ¥ is worth $0.01.

Notation: $x(t), t \geq 0$
↕ the exchange rate from FC to DC at time-t

Example. SIMPLE investment in FC.

@ time-0:
- BUY 1 unit of FC
  → the initial cost in DC is $x(0)$
- DEPOSIT the 1 unit of FC @ the "foreign" cont-comp, risk-free interest rate r_F

@ time-T:
- WITHDRAW the balance, i.e., $e^{r_F T}$ units of FC
- EXCHANGE this amt back to DC; get: $x(T)e^{r_F T}$ ← rnd var.

Analogy:

cont-div-paying stocks $\longleftrightarrow$ "foreign" currency

δ (dividend yield) $\longleftrightarrow$ $\boxed{r_F}$

Example.

- Prepaid forward price:

$$F^P_{0,T}(x) = x(0)e^{-r_F T}$$

- Forward price: r_D... cont.-compounded, risk-free interest rate for DC

$$F_{0,T}(x) = x(0)e^{(r_D - r_F)T}$$

- European:

	payoff	
call	$(x(T)-K)_+$	w/ exercise date T and the strike price K
put	$(K-x(T))_+$	

$V_C(0,x,K)$, $V_P(0,x,K)$ initial prices of the call/put — DC denominated

Put-call parity

$$V_C(0,x,K) - V_P(0,x,K) = x(0)e^{-r_F T} - Ke^{-r_D T}$$

Special Symmetry

$V_c(t, x, K)$ — K: strike price; x: the underlying exchange rate FC ↦ DC; t: valuation date (0 or T)

$V_c(0, x, K)$: initial price

$V_c(T, x, K)$: payoff

THE PAYOFF:

$$V_c(T, x, K) = (x(T) - K)_+$$

$$= x(T) \cdot K \cdot \left(\frac{1}{K} - \frac{1}{x(T)}\right)_+$$

- $\frac{1}{K}$: a FC-denominated strike
- $\frac{1}{x(T)}$: the reciprocal exchange rate
- $x(T)$: exchange rate FC ↦ DC at time-T
- K: a constant
- $\left(\frac{1}{K} - \frac{1}{x(T)}\right)_+$: PAYOFF of a put on DC w/ strike $1/K$

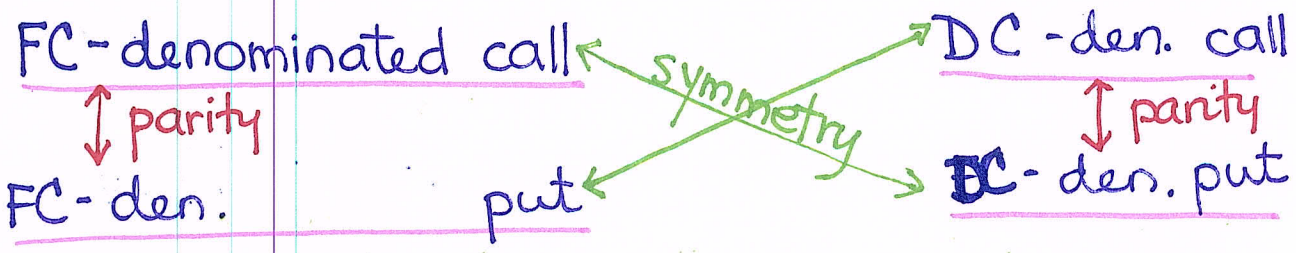
$$V_c(T, x, K) = x(T) \cdot K \cdot V_p\left(T, \frac{1}{x}, \frac{1}{K}\right)$$

$\Downarrow$ no arbitrage

$$V_c(0, x, K) = x(0) \cdot K \cdot V_p\left(0, \frac{1}{x}, \frac{1}{K}\right)$$

Put-call symmetry/duality

$$V_p(0, x, K) = x(0) \cdot K \cdot V_c\left(0, \frac{1}{x}, \frac{1}{K}\right)$$


FC - denominated call
parity
FC - den. put
symmetry
DC - den. call
parity
DC - den. put