vanilla calls/puts
"stocks for cash"

Currency options
"cash #1 for cash #2"

"stock #1 for stock #2"

Exchange options
Call / Put w/ BOTH the underlying and strike being RISKY ASSETS.

European

Vanilla
CALL @ time T:
The owner can pay K in return 1 unit of the underlying S

Exchange
CALL @ time T:
The owner can give up 1 unit of the STRIKE ASSET Q and in return get 1 unit of the underlying S

Payoff:
\[ V_c(T) = (S(T) - K)_+ \]

Payoff:
\[ V_{Ec}(T, S, Q) = (S(T) - Q(T))_+ \]
[PUT] @ time-T:
the owner of the option can
give up 1 unit of the underlying assets
and in return
get 1 unit of the strike asset $Q$

$V_{EP}(T, S, Q) = (Q(T) - S(T))^+$

Symmetry: $V_{EC}(T, S, Q) = V_{EP}(T, Q, S)$
\(\downarrow\) no arbitrage

$V_{EC}(t, S, Q) = V_{EP}(t, Q, S)$ for all $t \in [0, T]$
Maximum Option

\[ V_{\text{MAX}}(T) = \max[\mathcal{S}(T), \alpha(T)] \]

\[ \alpha: \text{Physical implementation} \]

**Properties:**

\[ V_{\text{MAX}}(T) \geq \mathcal{S}(T) \quad \Rightarrow \quad V_{\text{MAX}}(0) \geq \max(F^p_{0,T}(\mathcal{S}), F^p_{0,T}(\alpha)) \]

Can be perceived as costly?

Recall: For any \( a, b \):

\[ a + (b-a)^+ = \max(a, b) = b + (a-b)^+ \]

\[ \Rightarrow \quad V_{\text{MAX}}(T) = \max(\mathcal{S}(T), \alpha(T)) \equiv \alpha(T) + (\mathcal{S}(T) - \alpha(T))^+ \]

Max. option has a replicating portfolio consisting of:

- Exchange call w/ underlying \( \mathcal{S} \) and the strike asset \( \alpha \)
- [prepaid forward on \( \alpha \)]

No arbitrage?

\[ \Rightarrow \quad V_{\text{MAX}}(0) = F^p_{0,T}(\alpha) + V_{\text{EC}}(0, \mathcal{S}, \alpha) \quad (i) \]
\[ F_0^P(S) + V_{EC}(0, Q, S) = F_0^P(Q) + V_{EP}(0, Q, S) = F_0^P(S) + V_{EP}(0, S, Q) \] (iv)

\[ V_{EC}(0, S, Q) - V_{EP}(0, S, Q) = F_0^P(S) - F_0^P(Q) \]

Generalized PUT-CALL Parity
6. Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price.

\[ S_j(t) \] denotes the price of one share of stock \( j \) at time \( t \).

Consider a claim maturing at time 3. The payoff of the claim is

\[ \text{Maximum } (S_1(3), S_2(3)). \]

You are given:

(i) \( S_1(0) = $100 \)

(ii) \( S_2(0) = $200 \)

(iii) Stock 1 pays dividends of amount \( 0.05 S_1(t) \) \( dt \) between time \( t \) and time \( t + dt \).

(iv) Stock 2 pays dividends of amount \( 0.1 S_2(t) \) \( dt \) between time \( t \) and time \( t + dt \).

(v) The price of a European option to exchange Stock 2 for Stock 1 at time 3 is $10.

Calculate the price of the claim.

(A) $96

(B) $145

(C) $158

(D) $200

(E) $234