vanilla calls/puts

"stocks for cash"

currency options

"cash #1 for cash #2"

symmetry :)

"stock #1 for stock #2"

EXCHANGE OPTIONS

Call / Put w/ BOTH the underlying and strike being RISKY ASSETS.

EUROPEAN

Vanilla

CALL @ time-T:

the owner can

pay K

in return

1 unit of the underlying S

Exchange

CALL @ time-T:

the owner can

give up 1 unit of the STRIKE ASSET Q

and in return

get 1 unit of the underlying S

Payoff:

$$V_C(T) = (S(T) - K)_+$$

valuation date

Payoff:

the strike asset

$$V_{EC}(T, S, Q) = (S(T) - Q(T))_+$$

the underlying asset

$\boxed{\text{PUT}}$ @ time-T:

the owner of the option $\boxed{\text{can}}$ <u>give up</u> 1 unit of the underlying asset S and <u>in return</u> <u>get</u> 1 unit of the strike asset Q

$$\boxed{V_{EP}(T, S, Q) = (Q(T) - S(T))_+}$$

<u>Symmetry</u>: $V_{EC}(T, S, Q) = V_{EP}(T, Q, S)$

$\Downarrow$ no arbitrage

$$\boxed{V_{EC}(t, S, Q) = V_{EP}(t, Q, S) \quad \text{for all } t \in [0, T]}$$

Maximum Option

$$V_{MAX}(T) = MAX[S(T), Q(T)]$$

Q: Physical implementation!?

Properties:

$$\left.\begin{array}{l} V_{MAX}(T) \geq S(T) \\ V_{MAX}(T) \geq Q(T) \end{array}\right\} \Rightarrow V_{MAX}(0) \geq \max\left(F^P_{0,T}(S), F^P_{0,T}(Q)\right)$$

Can be percieved as costly!

Recall: For any a,b:

$$a + (b-a)_+ = \max(\underset{S(T)}{a}, \underset{Q(T)}{b}) = b + (a-b)_+$$

$$\Rightarrow V_{MAX}(T) = MAX(S(T), Q(T)) = Q(T) + (S(T) - Q(T))_+$$

max. option has a

REPLICATING PORTFOLIO consisting of:

- exchange call w/ underlying S and the strike asset Q
- prepaid forward on Q

no arbitrage!

$$\Rightarrow V_{MAX}(0) = F^P_{0,T}(Q) + V_{EC}(0, S, Q) \quad (i)$$

$$= F^P_{0,T}(S) + V_{EC}(0, Q, S)$$

$$= F^P_{0,T}(Q) + V_{EP}(0, Q, S)$$

$$= F^P_{0,T}(S) + \underbrace{V_{EP}(0, S, Q)}_{} \quad \text{(iv)}$$

(i) and (iv) $\Rightarrow$

$$\Rightarrow \boxed{V_{EC}(0,S,Q) - V_{EP}(0,S,Q) = F^P_{0,T}(S) - F^P_{0,T}(Q)}$$

Generalized PUT-CALL Parity

6. Consider a model with two stocks. Each stock pays dividends continuously at a rate proportional to its price.

$S_j(t)$ denotes the price of one share of stock j at time t.

Consider a claim maturing at time 3. The payoff of the claim is

$$\text{Maximum } \left(S_1(3), S_2(3)\right).$$

You are given:

(i) $S_1(0) = \$100$

(ii) $S_2(0) = \$200$

$\delta_1 = 0.05$

(iii) Stock 1 pays dividends of amount $\boxed{(0.05)}S_1(t)\,dt$ between time t and time $t+dt$.

(iv) Stock 2 pays dividends of amount $\boxed{(0.1)}S_2(t)\,dt$ between time t and time $t+dt$.

$\delta_2 = 0.10$

(v) $\boxed{\text{The price}}$ of a European option to exchange Stock 2 for Stock 1 at time 3 is $\boxed{\$10}$.

exchange call give up stock #2 ← strike
get stock #1 ← underlying

Calculate the price of the claim.

time-0 price is \$10

(A) \$96

(B) \$145

$\boxed{\text{(C) } \$158}$

(D) \$200

(E) \$234