Properties of put prices as functions of the strike $K$

Put-option prices are **increasing** as functions of the strike.

**Example.** Assume, to the contrary, that there are $K_1 < K_2$ such that $V_p(K_1) > V_p(K_2)$

I. $\checkmark$

II. Propose an arbitrage portfolio:
   - **Write** the $K_1$-strike put
   - **Long** the $K_2$-strike put

III. Verification:

\[
\text{PAYOFF} = \begin{cases} 
  + & \text{for } S < K_1 \\
  0 & \text{for } K_1 \leq S \leq K_2 \\
  - & \text{for } S > K_2 
\end{cases}
\]

\[
\text{PROFIT} = \text{PAYOFF} - \text{FV(Init. Cost)} > 0
\]

A short position w.r.t. the underlying $\Rightarrow$ An appropriate hedge for a long position in the underlying.

\[\Rightarrow \Leftarrow\]
Call bear spreads

\[ k_1 \leq k_2 \]

\[
\begin{cases}
\text{WRITE} & \text{the } k_1\text{-strike call} \\
\text{LONG} & \text{the } k_2\text{-strike call}
\end{cases}
\]

Tasks:
\begin{itemize}
  \item Draw the payoff curve.
  \item Compare the profits of the call bear spread and put bear spread.
\end{itemize}
Bounds on the growth

\[ K_1 < K_2 \]

\[ V_C(K_1) - V_C(K_2) \leq PV_{0,T}(K_2 - K_1) \]

\[ V_P(K_2) - V_P(K_1) \leq PV_{0,T}(K_2 - K_1) \]

arbitrage portfolio

CALL BEAR SPREAD

PUT BULL SPREAD

Verify @ home!
12. You are given:

(i) $C(K, T)$ denotes the current price of a $K$-strike $T$-year European call option on a non-dividend-paying stock.

(ii) $P(K, T)$ denotes the current price of a $K$-strike $T$-year European put option on the same stock.

(iii) $S$ denotes the current price of the stock. $S(0) = S$

(iv) The continuously compounded risk-free interest rate is $r$.

Which of the following is (are) correct?

(I) $0 \leq C(50, T) - C(55, T) \leq 5e^{-rT}$

(II) $50e^{-rT} \leq P(45, T) - C(50, T) + S \leq 55e^{-rT}$

(III) $45e^{-rT} \leq P(45, T) - C(50, T) + S \leq 50e^{-rT}$

(A) (I) only

(B) (II) only

(C) (III) only

(D) (I) and (II) only

(E) (I) and (III) only

$\sqrt{\text{bound on growth}}$

Put prices are increasing w.r.t. the strike.

$\Rightarrow$

$45e^{-rT} \leq V_p(45) - V_p(50) + 50e^{-rT} \leq 50e^{-rT}$

$\square$ is True!
Example. Assume, to the contrary, that there are $k_1 < k_2$ such that

$$V_p(k_2) - V_p(k_1) > PV_{0,T}(k_2 - k_1)$$

I. \( \checkmark \)

II. Propose our arbitrage portfolio!

Rewrite \( \circledast \) as:

$$V_p(k_2) > V_p(k_1) + PV_{0,T}(k_2 - k_1)$$

- LONG the $k_1$-strike put
- SHORT the $k_2$-strike put

III. Verification.

Init. Cost. = $V_p(k_1) - V_p(k_2) < -PV_{0,T}(k_2 - k_1)$.

**PAYOFF:**

$$\text{Profit (Put bull spread)} \geq k_1 - k_2 + FY(PV(k_2 - k_1)) = 0$$