OCT 26TH, 2015

Properties of put prices as functions of the strike K

Put-option prices are <u>INCREASING</u> as functions of the strike.

<u>Example</u>. Assume, to the contrary, that there are $K_1 < K_2$ such that $\boxed{V_p(K_1) > V_p(K_2)}$

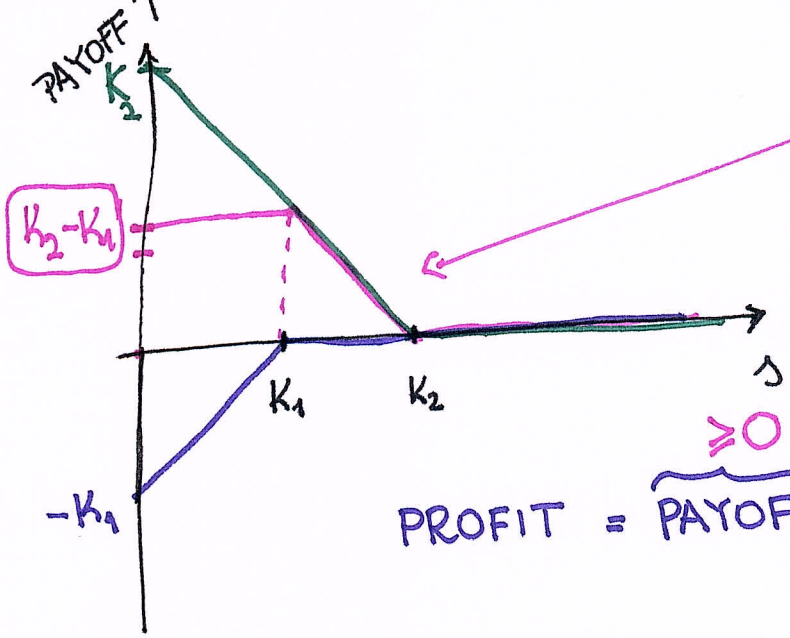
I. ✓

II. Propose an arbitrage portfolio:

- <u>WRITE</u> the <u>K_1-strike</u> put
- <u>LONG</u> the <u>K_2-strike</u> put

(PUT) BEAR SPREAD

III. Verification:



A <u>short</u> position w.r.t. the underlying

$\Rightarrow$ An appropriate HEDGE for a <u>long</u> position in the underlying.

$$\text{PROFIT} = \overbrace{\text{PAYOFF}}^{\geq 0} - \overbrace{FV(\text{Init. Cost})}^{<0} > 0$$

$\Rightarrow\Leftarrow$

Call bear spreads

$K_1 \leq K_2$

- **WRITE** the K_1-strike call
- **LONG** the K_2-strike call

Tasks:
- Draw the payoff curve.
- Compare the profits of the call bear spread and put bear spread.

Bounds on the growth

$K_1 < K_2$

$V_C(K_1) - V_C(K_2) \leq PV_{0,T}(K_2 - K_1)$

$V_P(K_2) - V_P(K_1) \leq PV_{0,T}(K_2 - K_1)$

arbitrage portfolio
CALL BEAR SPREAD ✓
PUT BULL SPREAD ✓

Verify @ home!

12. You are given:

(i) $C(K, T)$ denotes the current price of a K-strike T-year European call option on a nondividend-paying stock. $V_c(K)$

(ii) $P(K, T)$ denotes the current price of a K-strike T-year European put option on the same stock. $V_p(K)$

(iii) S denotes the current price of the stock. $S(0) = S$

(iv) The continuously compounded risk-free interest rate is r.

Which of the following is (are) correct?

$V_c(50) - V_c(55) \le PV_{0,T}(55-50) = 5e^{-rT}$

decrease

T (I) $0 \le C(50, T) - C(55, T) \le 5e^{-rT}$

(II) $50e^{-rT} \le P(45, T) - C(50, T) + S \le 55e^{-rT}$

(III) $45e^{-rT} \le P(45, T) - C(50, T) + S \le 50e^{-rT}$

$\Downarrow$ ~~B.~~ ~~D.~~

(A) (I) only

(B) (II) only

(C) (III) only

(D) (I) and (II) only

(E) (I) and (III) only

$V_p(45) - V_c(50) + S(0) = \left[\begin{array}{c}\text{Put-Call}\\ \text{Parity}\end{array}\right]$

$= V_p(45) - (V_p(50) + F^P_{0,T}(S) - PV_{0,T}(50))$

$+\cancel{S(0)}$ $\quad \cancel{=S(0)}$ no div.

$= V_p(45) - V_p(50) + 50e^{-rT}$

bound on growth

$-5e^{-rT} \le V_p(45) - V_p(50) \le 0$ Put prices are increasing w.r.t. the strike.

$\Rightarrow$ $/ + 50e^{-rT}$

$45e^{-rT} \le V_p(45) - V_p(50) + 50e^{-rT} \le 50e^{-rT} \Rightarrow$

III is True!

$\Downarrow$

E.

Example. Assume, to the contrary, that there are $K_1 < K_2$ such that

$$V_P(K_2) - V_P(K_1) > PV_{0,T}(K_2 - K_1) \quad] \circledast$$

I. ✓

II. Propose our arbitrage portfolio!

Rewrite $\circledast$ as:

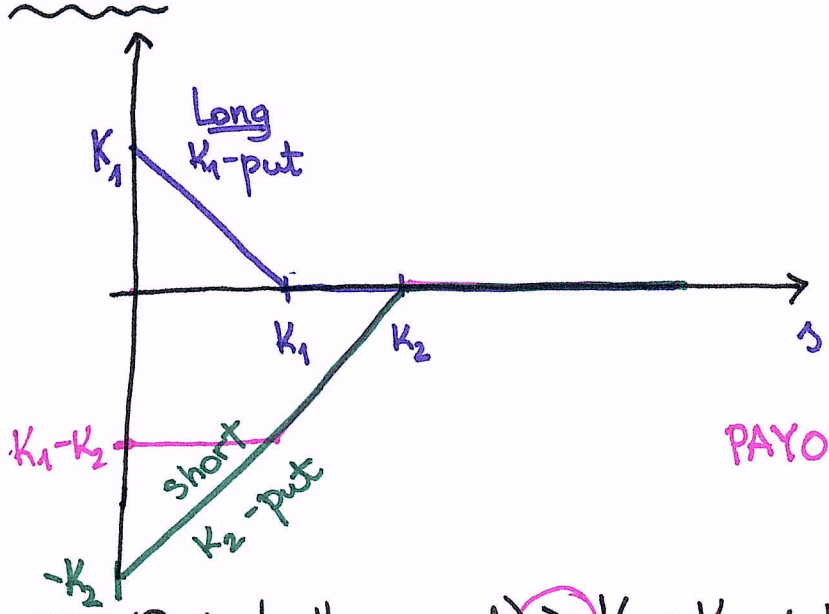
$$\underbrace{V_P(K_2)} > \underbrace{V_P(K_1)} + PV_{0,T}(K_2 - K_1)$$

- **LONG** the K_1-strike put
- **SHORT** the K_2-strike put

III. Verification.

$$\text{Init. Cost.} = V_P(K_1) - V_P(K_2) < -PV_{0,T}(K_2 - K_1).$$

PAYOFF:



PAYOFF $\geq K_1 - K_2$.

$$\text{PROFIT (Put bull spread)} > K_1 - K_2 + FV(PV(K_2 - K_1)) = 0$$