Put-call parity

Portfolio A:

\[ V_A(T) = V_c(T) - V_p(T) \]
\[ = (S(T) - K)_+ - (K - S(T))_+ \]
\[ = \begin{cases} 
S(T) - K & \text{if } S(T) \geq K \\
-(K - S(T)) & \text{if } S(T) < K 
\end{cases} \]

\[ \Rightarrow V_A(T) = S(T) - K \]

Portfolio B:

\{ \begin{align*}
& \text{long prepaid forward on } S \\
& \text{w/ delivery @ time } T \\
& \text{borrow } PV_{0,T}(K) \text{ @ the risk-free interest rate to be repaid @ time } T
\end{align*} \}

\[ V_B(T) = S(T) - K \]
\[ V_A(T) = S(T) - K = V_B(T) \]

\[ \implies V_A(0) = V_B(0) \]

\[ \implies V_C(0) - V_P(0) = F_{0,T}^p(S) - PV_{0,T}(K) \]

Put-Call Parity
Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60. \( S(0) = 60 \)

(ii) The call option currently sells for 0.15 more than the put option. \( V_c(0) - V_p(0) = 0.15 \)

(iii) Both the call option and put option will expire in 4 years. \( T = 4 \)

(iv) Both the call option and put option have a strike price of 70. \( K = 70 \)

Calculate the continuously compounded risk-free interest rate.

Put-call parity:

\[
V_c(0) - V_p(0) = \frac{F_{0,T}^p(S) - PV_{0,T}(K)}{S(0)}
\]

(A) 0.039
(B) 0.049
(C) 0.059
(D) 0.069
(E) 0.079

0.15 = S(0) - Ke^{-rT}

\[Ke^{-rT} = S(0) - 0.15 \quad \therefore K\]

\[e^{-rT} = \frac{S(0) - 0.15}{K}\]

\[-rT = \ln\left(\frac{S(0) - 0.15}{K}\right)\]

\[r = -\frac{1}{T} \ln\left(\frac{S(0) - 0.15}{K}\right) = -\frac{1}{4} \ln\left(\frac{60 - 0.15}{70}\right)\]

\[r = \ldots = 0.039 \quad \Rightarrow \text{A.}\]
1. On April 30, 2007, a common stock is priced at $52.00. You are given the following:

(i) Dividends of equal amounts will be paid on June 30, 2007 and September 30, 2007.

(ii) A European call option on the stock with strike price of $50.00 expiring in six months sells for $4.50.

(iii) A European put option on the stock with strike price of $50.00 expiring in six months sells for $2.45.

(iv) The continuously compounded risk-free interest rate is 6%.

Calculate the amount of each dividend.

\[
\text{Put-call parity} \\
V_c(0) - V_p(0) = F_{0,T}^p(S) - PV_{0,T}(K)
\]

(A) $0.51
(B) $0.73
(C) $1.01
(D) $1.23
(E) $1.45

\[
V_c(0) - V_p(0) = \frac{S(0)}{e^{-r \cdot \frac{1}{6}}} - De^{-r \cdot \frac{5}{12}} - 50e^{-r \cdot \frac{1}{2}}
\]

\[
\Rightarrow D = \frac{52 - 50e^{-0.03} - 4.50 + 2.45}{e^{-0.01} + e^{-0.025}} \approx 0.73 \Rightarrow \boxed{B}
\]
Continuous dividends

\[ V_c(0) - V_p(0) = S(0)e^{-ST} - PV_{0,T}(K) \]
Sample FM (DM)

53.

For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year 400-strike European call option costs 110. The annual risk-free rate of interest is a constant 6.5%.

\[ F_{0,4} = 300 \quad K = 400 \quad V_C(0) = 110 \quad r = 0.065 \]

Continuously compounded interest rate

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

\[ V_p(0) = ? \]

(A) 10.00
(B) 32.89
(C) 118.42
(D) 187.11
(E) 210.00

Put-call Parity

\[ V_C(0) - V_p(0) = \frac{F_{0,T}^P}{1} - PV_{0,T}(K) \]

\[ = PV_{0,T}(F_{0,T}) - PV_{0,T}(K) \]

\[ = (F_{0,T} - K)e^{-rT} \]

\[ V_p(0) = V_C(0) + (K - F_{0,T})e^{-rT} \]

\[ = 110 + (400 - 300)e^{-0.065 \cdot 4} = \ldots = 187.11 \]
Long Call + Short Put = "synthetic" forward

K ... not necessarily the no-arbitrage forward price