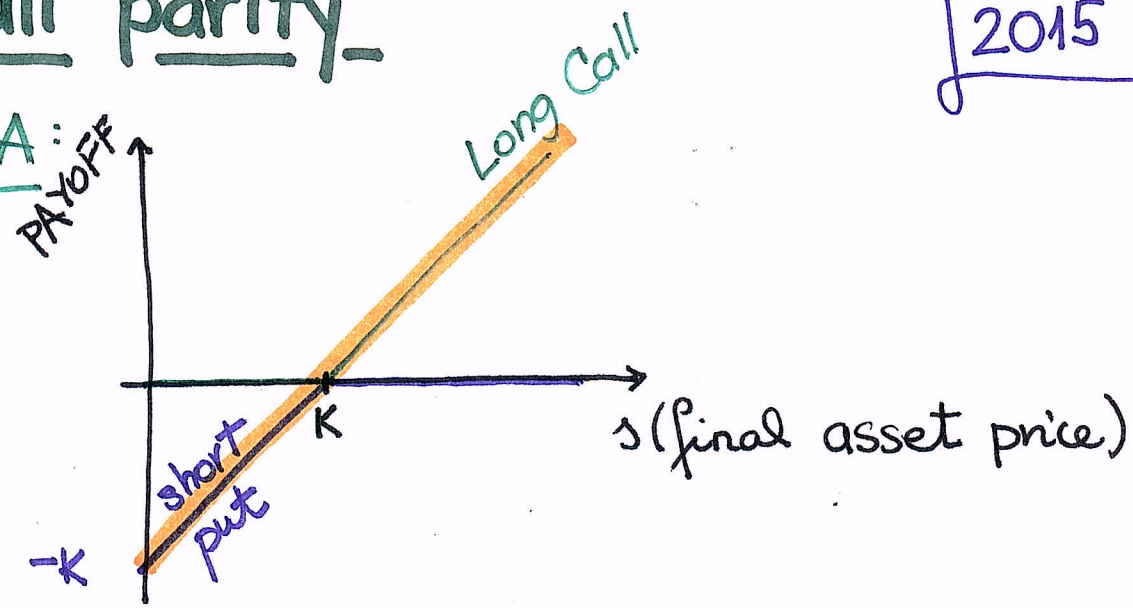
# Put-call parity

Oct 5th 2015

## Portfolio A:




$$V_A(T) = V_C(T) - V_P(T)$$

$$= (S(T)-K)_+ - (K-S(T))_+$$

$$= \begin{cases} S(T)-K & \text{if } S(T) \geq K \\ -(K-S(T)) & \text{if } S(T) < K \end{cases}$$

$$\Rightarrow \boxed{V_A(T) = S(T) - K}$$

## Portfolio B:

- long prepaid forward on S w/ delivery @ time-T
- borrow $PV_{0,T}(K)$ @ the risk-free interest rate to be repaid @ time-T

$$\boxed{V_B(T) = S(T) - K}$$

$$V_A(T) = S(T) - K = V_B(T)$$

$$\Rightarrow \quad \boxed{V_A(0) = V_B(0)}$$

$$\Rightarrow \quad \boxed{V_C(0) - V_P(0) = F^P_{0,T}(S) - PV_{0,T}(K)}$$

<u>Put-Call Parity</u>

Sample MFE

1. Consider a European call option and a European put option on a nondividend-paying stock. You are given:

(i) The current price of the stock is 60. $S(0) = 60$

(ii) The call option currently sells for 0.15 more than the put option. $V_c(0) - V_p(0) = 0.15$

(iii) Both the call option and put option will expire in 4 years. $T = 4$

(iv) Both the call option and put option have a strike price of 70. $K = 70$

Calculate the continuously compounded risk-free interest rate. $r = ?$

(A) 0.039

(B) 0.049

(C) 0.059

(D) 0.069

(E) 0.079

Put-call parity

$$\underbrace{V_c(0) - V_p(0)}_{0.15} = \underbrace{F^P_{0,T}(S)}_{= S(0)} - \overbrace{PV_{0,T}(K)}^{= Ke^{-rT}}$$

$=$ ← no dividends!

$$0.15 = S(0) - Ke^{-rT}$$

$$Ke^{-rT} = S(0) - 0.15 \quad /:K$$

$$e^{-rT} = \frac{S(0) - 0.15}{K}$$

$$-rT = \ln\left(\frac{S(0) - 0.15}{K}\right)$$

$$r = -\frac{1}{T}\ln\left(\frac{S(0) - 0.15}{K}\right) = -\frac{1}{4}\ln\left(\frac{60 - 0.15}{70}\right)$$

$$r = \dots = 0.039 \Rightarrow A.$$

**BEGINNING OF EXAMINATION**
ACTUARIAL MODELS – FINANCIAL ECONOMICS SEGMENT

**1.** On April 30, 2007, a common stock is priced at \$52.00. You are given the following:

(i) Dividends of equal amounts will be paid on June 30, 2007 and September 30, 2007.

(ii) A European call option on the stock with strike price of \$50.00 expiring in six months sells for \$4.50.

(iii) A European put option on the stock with strike price of \$50.00 expiring in six months sells for \$2.45.

(iv) The continuously compounded risk-free interest rate is 6%. $\boxed{r=0.06}$

Calculate the amount of each dividend.

(A) \$0.51

(B) \$0.73

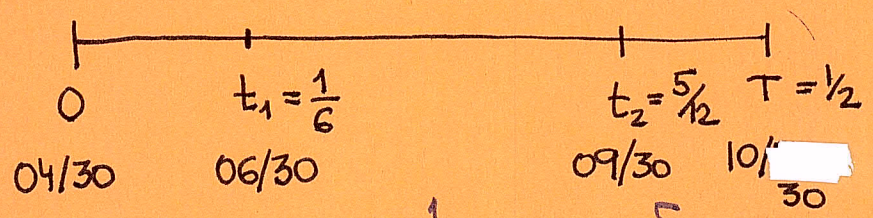
(C) \$1.01

(D) \$1.23

(E) \$1.45

Put-call parity

$$V_c(0) - V_p(0) = \boxed{F^P_{0,T}(S)} - PV_{0,T}(K)$$




$$V_c(0) - V_p(0) = S(0) - De^{-r\cdot\frac{1}{6}} - De^{-r\cdot\frac{5}{12}} - 50e^{-r\cdot\frac{1}{2}}$$

$$\Rightarrow D = \frac{52 - 50e^{-0.03} - 4.50 + 2.45}{e^{-0.01} + e^{-0.025}} \approx 0.73 \Rightarrow \text{B.}$$

# Continuous dividends

$$\boxed{V_C(0) - V_P(0) = S(0)e^{-\delta T} - PV_{0,T}(K)}$$

Sample FM (DM)

53.

$F_{0,4} = 300$

For each ton of a certain type of rice commodity, the four-year forward price is 300. A four-year 400-strike European call option costs 110. $K = 400$ $V_c(0) = 110$

$T=4$

The annual risk-free ~~force of interest~~ is a constant 6.5%. $r = 0.065$

continuously compounded interest rate

Calculate the cost of a four-year 400-strike European put option for this rice commodity.

$V_p(0) = ?$

(A) 10.00

(B) 32.89

(C) 118.42

(D) 187.11

(E) 210.00

Put-call Parity

$$V_c(0) - V_p(0) = \underbrace{F^P_{0,T}} - PV_{0,T}(K)$$

$$= PV_{0,T}(F_{0,T}) - PV_{0,T}(K)$$

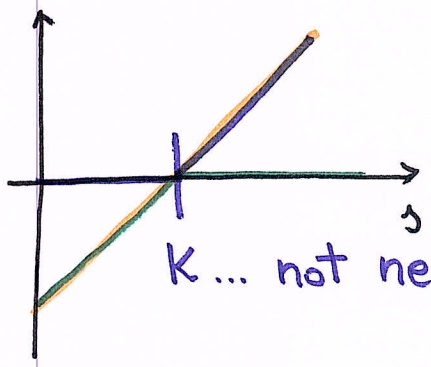
$$= (F_{0,T} - K)e^{-rT}$$

$$V_p(0) = V_c(0) + (K - F_{0,T})e^{-rT}$$

$$= 110 + (400 - 300)e^{-0.065 \cdot 4} = \cdots = 187.11$$

D.

Long Call + Short Put = "synthetic" forward




K ... not necessarily the no-arbitrage forward price