12.1. **Gap calls.** A European *gap call option* is a derivative security on an underlying asset (with price denoted by $S = \{S(t), t \geq 0\}$) which given:
- an exercise date $T$;
- a strike price $K_s$;
- a trigger price $K_t$.

provides the payoff

$$V_{GC}(T) = (S(T) - K_s)1_{S(T) \geq K_t}$$

to its owner.

**Problem 12.1.** Consider a gap call option with $K_s \leq K_t$.
- Draw its payoff curve.
- Is a long gap call a long or a short position with respect to the underlying asset for the above ordering of the strike price and the trigger price?

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**INSTRUCTOR:** Milica Ćudina
Problem 12.2. Consider a gap call option with $K_f < K_s$.
- Draw its payoff curve.
- Do you think that the word "option" is entirely appropriate in this case?

Problem 12.3. Create a replicating portfolio for the gap call option consisting of cash-or-nothing call options and asset-or-nothing call options.
12.2. **Gap puts.** A European *gap put option* is a derivative security on an underlying asset (with price denoted by \( S = \{ S(t), t \geq 0 \} \) which given:

- an exercise date \( T \);
- a *strike* price \( K_s \);
- a *trigger* price \( K_t \)

provides the payoff

\[
V_{gap}(T) = (K_s - S(T))I_{S(T) < K_t}
\]

to its owner.

**Problem 12.4.** Consider a gap put option with \( K_s \leq K_t \). Draw its payoff curve.
Problem 12.5. Consider a gap put option with $K_s > K_t$. Draw its payoff curve.

\[
V_{GP}(T) = (K_s - S(T)) \mathbb{I}_{[S(T) < K_t]} = K_s \cdot V_{CP}(T) - V_{AP}(T)
\]

\[
V_{CP}(T) = \mathbb{I}_{[S(T) < K_t]}
\]
\[
V_{AP}(T) = S(T) \mathbb{I}_{[S(T) < K_t]}
\]

\[
\downarrow \quad \text{Long}
\]
\[
\cdot \quad K_s \quad \text{cashputs w/ trigger } K_t
\]
\[
\cdot \quad \text{one short asset put w/ trigger } K_t
\]

Problem 12.6. Create a replicating portfolio for the gap put option consisting of cash-or-nothing put options and asset-or-nothing put options.
12.3. Put-call parity for gap options.

Problem 12.7. Consider the following portfolio:

- one long gap call option with trigger price $K_t$ and the strike price $K_s$,
- one short otherwise identical gap put option.

(i) What is the initial cost of the above portfolio expressed in terms of the price of the gap call $V_{GC}(0)$ and the price of the gap put $V_{GP}(0)$?

(ii) What is the payoff of the above portfolio?

(iii) Based on your answers to the above two questions, what is put-call parity for gap options?

- Initial cost: $V_{GC}(0) - V_{GP}(0)$

- Payoff: $V_{GC}(T) - V_{GP}(T) = $

\[= (S(T) - K_s)\mathbb{1}_{[S(T) \geq K_t]} - (K_s - S(T))\mathbb{1}_{[S(T) < K_t]}\]

\[= S(T) - K_s\]

the trigger price is gone ....

- $V_{GC}(0) - V_{GP}(0) = F^p_{oT}(S) - PV_{oT}(K_s)$

Put-call parity.
Problem. \( V_{cc}(0) + V_{cp}(0) = ? \)

A bond (zero-coupon) redeemable @ time-\( T \) for $1 can be replicated \(
\left\{
\begin{array}{c}
\text{long cash call} \\
\text{long cash put}
\end{array}
\right.
\)

\( \Rightarrow \quad V_{cc}(0) + V_{cp}(0) = PV_{0T}(1) = e^{-rT} \)

Problem. \( V_{ac}(0) + V_{ap}(0) = K = F_{0T}^P(S) \)

draw the payoff curve.