**TRUE/FALSE**

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**MULTIPLE CHOICE**

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**FOR THE GRADER’S USE ONLY:**

<table>
<thead>
<tr>
<th>Def’n</th>
<th>T/F</th>
<th>1.10</th>
<th>1.11</th>
<th>M.C.</th>
<th>Σ</th>
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</table>
1.1. **DEFINITIONS.**

**Problem 1.1.** (10 points) Write the definition of an arbitrage portfolio.

Rubric:

<table>
<thead>
<tr>
<th>Evaluation</th>
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<tbody>
<tr>
<td><strong>Just stating “riskless investment”.</strong></td>
</tr>
<tr>
<td><strong>Only: profit always non-negative.</strong></td>
</tr>
<tr>
<td>“Cashflows” or “it” in stead of “profit”.</td>
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<tr>
<td>“payoff” in stead of “profit”.</td>
</tr>
<tr>
<td>No mention of “payoff” or “profit”.</td>
</tr>
<tr>
<td>Not stating “strictly positive in at least one state of the world”, but mentioning strictly positive without qualifier.</td>
</tr>
<tr>
<td>Only “payoff always positive/non-negative”</td>
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<tr>
<td>Stating “at all time” or “at some point” instead of the events/states-of-the-world/scenarios.</td>
</tr>
<tr>
<td>Stating “increasing” or “decreasing” instead of the positive/non-negative.</td>
</tr>
</tbody>
</table>
1.2. **TRUE/FALSE QUESTIONS.** *Please, circle the correct answer on the front page of this exam.*

**Problem 1.2.** (2 points)
The profit diagram and the payoff diagram for long positions in a forward contract are identical. *True or false?*

**Solution:** TRUE

**Problem 1.3.** (2 points)
A portfolio consisting of a short asset and a long put option is referred to as a floor. *True or false?*

**Solution:** FALSE

**Problem 1.4.** (2 points)
Derivatives were designed to comply with tax regulations. *True or false?*

**Solution:** FALSE

**Problem 1.5.** (2 points)
An in-the-money option is one which would have a positive payoff if exercised immediately. *True or false?*

**Solution:** TRUE

**Problem 1.6.** (2 points)
In an insurance market, individuals that do not incur losses have shared risk with individuals that do incur losses. *True or false?*

**Solution:** TRUE

**Problem 1.7.** (2 points)
A short put option has an unlimited loss potential. *True or false?*

**Solution:** FALSE

**Problem 1.8.** (2 points)
Consider a $70-strike European put and an $80-strike European put. If the $70-strike put is in-the-money, then the $80-strike put is also in-the-money. *True or false?*

**Solution:** TRUE

**Problem 1.9.** (2 points)
To cover a written call option, a market maker should short the underlying asset. *True or false?*
Solution: FALSE
1.3. **FREE-RESPONSE PROBLEMS.**

**Problem 1.10.** (20 points)

Consider a stock whose current price equals $50 per share. We observe the following prices of one-year European call options:

<table>
<thead>
<tr>
<th>Strike price</th>
<th>Option price</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>12.54</td>
</tr>
<tr>
<td>50</td>
<td>10.36</td>
</tr>
<tr>
<td>55</td>
<td>8.54</td>
</tr>
</tbody>
</table>

The continuously compounded risk-free interest rate is given to be 0.05.

Determine the interval of final asset prices such that the $55-strike call produce a higher profit than the $50-strike call, but a lower profit than the $45-strike call. Express your answer as an interval.

**Solution:**

We are seeking all $s$ such that

$$(s - 50)_+ - 10.36 e^{0.05} < (s - 55)_+ - 8.54 e^{0.05} < (s - 45)_+ - 12.54 e^{0.05}$$

First, let us focus on

$$(s - 50)_+ - 10.36 e^{0.05} < (s - 55)_+ - 8.54 e^{0.05}$$

The above inequality holds for all $s < 50$ and for no $s > 55$. For $50 \leq s \leq 55$, we have

$$s - 50 - 10.36 e^{0.05} < -8.54 e^{0.05} \iff s < 50 + 1.91 = 51.91$$

Second, we focus on

$$(s - 55)_+ - 8.54 e^{0.05} < (s - 45)_+ - 12.54 e^{0.05}$$

This inequality is true for all $s > 55$ and it is not true for any $s < 45$. For $45 \leq s \leq 55$, we have to find all $s$ such that

$$-8.54 e^{0.05} < s - 45 - 12.54 e^{0.05} \iff s > 45 + 4 e^{0.05} = 49.21.$$ 

Our final answer is $(49.21, 51.91)$.

**Rubric:**

The requested interval is:
Problem 1.11. (20 points)
A certain continuous-dividend-paying stock is currently priced at $100 and its dividend yield is given to be 0.02.

The continuously compounded risk-free interest rate is given to be 0.05.
You observe that the forward price of $F = 103$ for delivery of the above stock in 1 year is available in the market.

• Is there an arbitrage opportunity available? If your answer is affirmative, substantiate your suspicion. If your answer is negative, show why.

• If your answer was positive, propose an arbitrage portfolio.

• Verify that your proposed arbitrage portfolio is, indeed, an arbitrage portfolio.

Solution:
Diagnosis. The no-arbitrage forward price equals
\[ F_{0,T}(S) = 100e^{(0.05-0.02)1} = 103.045 \neq F = 103 \]
So, we conclude that there is an arbitrage opportunity.

Construction. We notice that the observed forward price exceeds the no-arbitrage forward price. So, we propose to construct an arbitrage portfolio as follows:

• long one observed forward contract,

• short-sell \( e^{-\delta T} = e^{-0.02} \) shares of stock.

Verification. The initial cost of the above portfolio is \(-100e^{-0.02}\). The net effect of closing the short state and fulfilling the obligations of the long forward result in a payoff of \(-F = -103\). Hence, the profit is
\[ -103 - (-100e^{-0.02} \times e^{0.05}) > 0. \]
Our portfolio is, indeed, an arbitrage portfolio.
1.4. **MULTIPLE CHOICE QUESTIONS.**

> Please, circle the correct answer on the front page of this exam.

**Problem 1.12.** (5 points) The *Supreme Sparkles* glitter-delivery company is contracted to sell 100,000 bushels of glitter this holiday season. As glitter occurs naturally in areas rich with unicos, pixies, and the more mild-mannered leprechauns, it simply gathered at no cost to the *Supreme Sparkles* company. However, they do need to pay a flat fee of one million dollars to the *Ilvermorny School of Witchcraft and Wizardry* on whose grounds the glitter is collected.

Since *Supreme Sparkles* sells glitter at a market price, they are exposed to risk. They hedge this risk with 100,000 **collars** on a bushel of glitter each. The collars are built using $12-dollar puts and $16-dollar calls.

Find the range of the **payoff** of their total **hedged portfolio**.

(a) $[-600,000, -200,000]$
(b) $[200,000, 600,000]$
(c) $[1,200,000, 1,600,000]$
(d) $[-200,000, 200,000]$
(e) None of the above.

**Solution: (b)**

Let us denote the market price of glitter by $S(T)$ (per bushel). The payoff is

$$100,000(S(T) + (12 - S(T))_+ - (S(T) - 16)_+ - 10).$$

The minimal payoff is $200,000 and the maximal payoff is $600,000.

**Problem 1.13.** (5 points) A jeweler buys platinum – the primary input needed for her stupendous creations. One ounce of platinum yields one unit of jewelry. The cost of all other inputs is negligible. The jeweler is contracted to sell each unit of jewelry for $925.

We model the market price of an ounce of platinum as taking one of the following three values: $900, $920, and $930. Their probabilities are modeled to be 0.3, 0.5, and 0.2 (in that order).

The jeweler hedges using a $915-strike call on the platinum. What is the jeweler’s total expected payoff?

(a) $10
(b) $14.50
(c) $910.50
(d) $926
(e) None of the above.

**Solution: (b)**

The jeweler’s payoff is

$$925 - S(T) + (S(T) - 915)_+ = 925 - \min(S(T), 915).$$

So, the expected payoff equals

$$925 - (900 \times 0.3 + 915 \times 0.7) = 925 - 910.50 = 14.50.$$
Problem 1.14. (5 points)
The current market price of Stock A is $50 per share. An agent buys a one-year, $50-strike put option on Stock A for $3.55. Calculate the agent’s maximum possible loss with the continuously compounded risk-free interest rate equal to 0.04.

(a) $3.41
(b) $3.55
(c) $3.69
(d) $46.31
(e) None of the above.

Solution: (c)
The maximum loss for a long put option occurs when the option is not exercised. We get

\[ 3.55e^{0.04} = 3.69. \]
Problem 1.15. (5 points) Source: Sample FM(DM) Problem #56.
Determine which of the following positions can have the same cash flows as a short stock position.
(a) A long forward contract and a long zero-coupon bond.
(b) A long forward contract and a short zero-coupon bond.
(c) A short forward contract and a long zero-coupon bond.
(d) A short forward contract and a short zero-coupon bond.
(e) A short forward contract and a short put option.
Solution: (d)

Problem 1.16. (5 points) The market Index is priced at $950. The index pays no dividends. The continuously compounded risk-free interest rate is 7%. What is the forward price for a forward contract with delivery date 9 months from today?
(a) $937.48
(b) $950.00
(c) $984.36
(d) $1001.21
(e) None of the above.
Solution: (d)

In our usual notation,
\[ F_{0,T}(S) = S(0)e^{(r-\delta)T} = 950e^{(0.07)0.75} \approx 1001.21 \]

Problem 1.17. The payoff of an cash-or-nothing put option with strike price \( K \) and exercise date \( T \) is given by:
(a) \( \mathbb{1}_{S(T) < K} \)
(b) \( \mathbb{1}_{S(T) \geq K} \)
(c) \( S(T)\mathbb{1}_{S(T) \geq K} \)
(d) \( (S(T) - K)\mathbb{1}_{S(T) \geq K} \)
(e) None of the above.
Solution: (a)

Problem 1.18. (5 points) A market index now sells for $1000 per unit and its dividend yield equals 0.01. The continuously compounded risk-free interest rate equals 0.05. Find the prepaid forward price for delivery of one unit of the above index in two years.
(a) $951.23
(b) $960.79
(c) $980.20
(d) $1000
(e) None of the above.
Solution: (c)

\[ F_{0.2}(S) = S(0)e^{-2t} = 1000e^{-0.02} = 980.199. \]