UNIVERSITY OF TEXAS AT AUSTIN

Problem set 1

Final exam prep.

Problem 1.1. (2 points) It is never optimal to exercise an American call option on a non-dividend paying stock early. *True or false?*

Problem 1.2. (2 points) *Source: MFE Exam, Spring 2009, #12.*
Let $V_C(K)$ and $V_P(K)$ denote the prices at time 0 of the European call and put options (respectively) both with maturity T. Denote the price at time 0 of the underlying asset by $S(0)$ and the constant continuously compounded interest rate by r. Then,

$$0 \leq V_C(50) - V_C(55) \leq 5e^{-rT}.$$

Problem 1.3. (2 pts) A (long) put is a short position with respect to the underlying asset price. *True or false?*

Problem 1.4. (5 points) A strategy consists of longing a put on the market index with a strike of \$830 and shorting a call option on the market index with a strike price of \$830. The put premium is \$18.00 and the call premium is \$44.00. Interest rates are 0.5% effective per month. Determine the net profit if the index price at expiration is \$830 (in 6 months).

(a) \$0
(b) \$23.67 loss
(c) \$26.79 gain
(d) \$28.50 gain
(e) None of the above.

Problem 1.5. The current price of a continuous-dividend-paying stock is \$100 per share. Its volatility is given to be 0.2 and its dividend yield is 0.03.

The continuously compounded risk-free interest rate equals 0.04.

Consider a \$95-strike European put option on the above stock with nine months to expiration. Using a three-period forward binomial tree, find the price of this put option.

(a) \$2.97
(b) \$3.06
(c) \$3.59
(d) \$4.32
(e) None of the above.

Problem 1.6. (5 points) *Source: SoA Sample IFM (Derivatives – Intro) Problem #30.*
You are trying to decide whether to use forward contracts or futures contracts when committing to buy an underlying asset at some date in the future. Which of the following is **NOT** a distinguishing characteristic of futures contracts, relative to forward contracts?

(a) Contracts are settled daily, and marked-to-market.
(b) Contracts are more liquid, as one can offset an obligation by taking the opposite position.
(c) Contracts are more customized to suit the buyer's needs.
(d) Contracts are structured to minimize the effects of credit risk.
(e) Contracts have price limits, beyond which trading may be temporarily halted.

Problem 1.7. Let $V_{C,C}(0)$ denote the price of a comopound call on a call. Let $V_{P,C}(0)$ denote the price of an otherwise identical comopound put on the same call option. Let $V_C(0)$ denote the price of the underlying call option. Let $S(0)$ denote the initial price of the underlying asset for this vanilla call option.

Which one(s) of the following inequalities is (are) **always** true for the above prices?

(a) $V_{C,C}(0) > V_{P,C}(0)$
(b) $V_{C,C}(0) - V_{P,C}(0) \leq V_C(0)$
(c) $V_{C,C}(0) > S(0)$
(d) $V_C(0) > S(0)$
(e) None of the above.

Problem 1.8. (5 points) Let $K_1 = 50, K_2 = 55$ and $K_3 = 65$ be the strikes of three European call options on the same underlying asset and with the same exipration date. Let $V_C(K_i)$ denote the price at time-0 of the option with strike K_i for $i = 1, 2, 3$.
We are given that $V_C(K_1) = 16$ and $V_C(K_3) = 1$. What is the maximum possible value of $V_C(K_2)$ which still does not violate the convexity property of option prices?

(a) $V_C(K_2) = 6$
(b) $V_C(K_2) = 8$
(c) $V_C(K_2) = 10$
(d) $V_C(K_2) = 11$
(e) None of the above.

Problem 1.9. (5 points) Your oil tanker is scheduled to take oil from port *Argosy* to port *Betelgeuse.* Which combination of options on oil would be the best hedge in this situation?

(a) Long a call in port *Argosy* and short a put in port *Betelgeuse.*
(b) Long a call in port *Argosy* and long a put in port *Betelgeuse.*
(c) Short a call in port *Argosy* and short a put in port *Betelgeuse.*
(d) Short a call in port *Argosy* and long a put in port *Betelgeuse.*
(e) None of the above.

Problem 1.10. (5 points) Consider the one-period binomial option pricing model. Let $V_C(0) > 0$ denote the price of a European call on a stock which pays continuous dividends. What is the impact on the value of European call option prices if the company decides to increase the dividend yield paid to the shareholders?

(a) The call option price will drop.
(b) The call option price will increase.
(c) The call option price will always remain constant.
(d) The impact on the price of the call cannot be determined using the binomial option pricing model.
(e) There is not enough information provided.

Problem 1.11. (5 points) Which one of the following positions always has an infinite upward potential in the sense that the payoff diverges to positive infinity as the argument s (standing for the final stock price) tends to positive infinity?

(a) A long call option.
(b) A long ratio spread.
(c) A long bull spread.
(d) A long butterfly spread.
(e) None of the above.

Problem 1.12. The current stock price is observed to be \$100 per share. The stock is projected to pay dividends continuously at the rate proportional to its price with the dividend yield of 0.03. The stock's volatility is given to be 0.23. You model the evolution of the stock price using a two-period forward binomial tree with each period of length one year.

The continuously compounded risk-free interest rate is given to be 0.04.

What is the price of a two-year, \$101-strike **American** put option on the above stock consistent with the above stock-price tree?

(a) About \$6.62
(b) About \$8.34
(c) About \$8.83
(d) About \$11.11
(e) None of the above.

Problem 1.13. (12 points) Consider a two-period binomial model for a non-dividend paying asset S with $S(0) = 50$ and $u = 1/d = 2$. Let $i = 0.25$ denote the effective interest rate per period. You need to price a European put option on S which expires at the end of the two periods and has the strike $K = 70$.

(i) (5 pts) Find the values of the given option at all the nodes in the binomial tree. In particular, find the no-arbitrage price at time 0 of this option.
(ii) (5 pts) Find the number of shares Δ one needs to invest in at every node in the tree in order to replicate the option.
(iii) (2 pts) If the option were American, would there be early exercise?

Problem 1.14. (10 points)

Consider a one-period forward binomial tree with $h = 1$, $S(0) = 100, r = 0.08, \sigma = 0.3, \delta = 0.08$.

(a) (5 pts) Find the expression $V_C^A(0, K)$ for the time-0 price of the American call option on S with strike K and maturity at the end of the period.
(b) (3 pts) Determine the condition for the strike K to be such that early exercise occurs?
(c) (2 pts) In particular, is there early exercise for $K = 70$?

Problem 1.15. (10 points) The current price of a non-dividend-paying stock is \$100 per share. A two-period binomial stock-price tree is used to model the movements of the stock price during the following year. The up and down factors are given to be $u = 1.2$ and $d = 0.9$.

The continuously compounded, risk-free interest rate equals 0.06.

Consider a \$110-strike, one-year American put on the above stock. Use the two-period binomial stock-price tree to calculate the current price of the American put.

Problem 1.16. (2 points) The price of a **up-and-in** option is increasing as a function of its barrier (with every other input held fixed).

Problem 1.17. (5 points) The current price of a certain stock is denoted by $S(0)$. Let the price of an **up-and-out** call option on that stock with barrier H be denoted by $V_C(0, H)$. Let the price of an otherwise identical vanilla call option be denoted by $V_C(0)$. With the limit is taken while all the other parameters are held the same, what is

$$\lim_{H \to \infty} V_C(0, H)$$

equal to?

(a) 0
(b) $V_C(0)$
(c) $S(0)$
(d) ∞
(e) None of the above.

Problem 1.18. (5 points) Consider a non-dividend paying stock whose current price is \$100 per share. Its volatility is given to be 0.30. You model the evolution of the stock price over the following year using a two-period forward binomial tree.

The continuously compounded risk-free interest rate is 0.05.

Consider a \$100-strike, one-year **knock-in** call option with a barrier of \$110 on the above stock. What is the price of this option consistent with the above stock-price model?

(a) About 11.55
(b) About 12.75
(c) About 13.96
(d) About 66.05
(e) None of the above.

Problem 1.19. (2 points) A time$-T$ exchange call with underlying $\mathbf{S}$ and strike asset $\mathbf{Q}$ is always worth strictly more than an exchange put option with underlying $\mathbf{Q}$ and strike asset $\mathbf{S}$. *True or false?*

Problem 1.20. (2 points) Gap put options always have a nonnegative payoff. *True or false?*

Problem 1.21. (2 points) A bear spread is a long position with respect to the underlying asset. *True or false?*

Problem 1.22. (2 points) In the binomial asset pricing model, the number of shares Δ of the underlying asset in the replicating portfolio for a **put** option is always positive. *True or false?*

Problem 1.23. (5 points) An investor short sells one share of a non-dividend-paying stock and buys an at-the-money, T-year, European call option on this stock. The call premium is denoted by $V_C(0)$. Assume that there are no transaction costs. The continuously compounded, risk-free interest rate is denoted by r. Let the argument s represent the stock price at time T.

(i) (3 points) Determine an algebraic expression for the investor's profit at expiration T in terms of $V_C(0), r, T$ and the strike K.
(ii) (2 points) In particular, how does the expression you obtained in (i) simplify if the call is in-the-money on the exercise date?

Problem 1.24. (10 pts) For a two-period binomial model, you are given that:

(1) each period is one year;
(2) the current price of a non-dividend-paying stock S is $S(0) = \$20$;
(3) $u = 1.2$, with u as in the standard notation for the binomial model;
(4) $d = 0.8$, with d as in the standard notation for the binomial model;
(5) the continuously compounded risk-free interest rate is $r = 0.04$.

Consider a **chooser** option such that its owner can decide after one year whether the option becomes a put or a call option with exercise date at time-2 and strike equal to \$20.

Find the price of the chooser option.

Problem 1.25. (5 points) A discrete-dividend-paying stock sells today for \$100 per share. The continuously compounded, risk-free interest rate is 0.04. The first dividend will be paid at in three months in the amount of \$2. The remaining dividends will be equal to \$1 and continue to be paid out quarterly. What is the **forward price** of this stock for delivery in seven months?

(a) \$73.02
(b) \$97.04
(c) \$99.33
(d) \$100
(e) None of the above.

Problem 1.26. In our usual notation, which of the parameter choices below creates a binomial model with an arbitrage opportunity?

(a) $u = 1.18, \quad d = 0.87, \quad r = 0.05, \quad \delta = 0, \quad h = 1/4$
(b) $u = 1.23, \quad d = 0.80, \quad r = 0.05, \quad \delta = 0.06, \quad h = 1/2$
(c) $u = 1.08, \quad d = 1, \quad r = 0.05, \quad \delta = 0.04, \quad h = 1$
(d) $u = 1.28, \quad d = 0.78, \quad r = \delta, \quad h = 2$
(e) None of the above.

Problem 1.27. The current price of a non-dividend-paying stock is \$50 per share. You observe that the price of a three-month, at-the-money American call option on this stock equals \$3.50.

The continuously compounded risk-free interest rate is 0.04.

Find the premium of the European three-month, at-the-money put option on the same underlying asset.

(a) About \$3
(b) About \$3.50
(c) About \$4
(d) About \$5.46
(e) None of the above.

Problem 1.28. Consider a non-dividend paying stock whose current price is \$95 per share. You model the evolution of this stock price over the following year using a one-period binomial tree under the assumption that the stock price can be either \$120, or \$75 in one year.

The continuously compounded risk-free interest rate is 0.06.

Consider a \$100-strike, one-year European **straddle** on the above stock. What is the straddle's price consistent with the above stock-price model?

(a) About \$10
(b) About \$10.83
(c) About \$15.45
(d) About \$20.84
(e) None of the above.