1.1. **Payoff and profit for a producer of goods who hedges using a forward contract.**

**Problem 1.1.** Consider the general case in which

- \( C \) stands for the total aggregate fixed and variable costs of production per unit of good;
- \( F \) stands for the forward price per unit of good.

What is the price \( s^* \) per unit of good at which the profit of a producer who hedges using a forward contract equals the profit of the producer who does not hedge at all?

**Solution:**

\[ s^* - C = F - C \quad \Rightarrow \quad s^* = F \]

**Problem 1.2.** A farmer produces one million bushels of corn. The total cost of production is \$1.3 million. The farmer entered a forward contract to hedge at a forward price of \$2.50 per bushel on one million bushels. What is the farmer’s profit?

**Solution:**

\[ 10^6(2.50 - 1.30) = 1.2 \times 10^6. \]

Let the farmer have a tax rate of 25%. What is his after-tax profit?

**Solution:**

\[ 0.75 \times 1.2 \times 10^6 = 800,000. \]

**Problem 1.3.** Assume that farmer Brown is uncertain about his crop yield. Based on past experience, he thinks the following is a good model:

- 100,000 bushels with probability 1/4;
- 80,000 bushels with probability 3/4.

How many forward contracts do you think farmer Brow should short to hedge against fluctuations in corn prices at harvest time? Explain your way of thinking . . .

**Solution:** This is an open-ended problem of sorts. Depending on the farmer’s attitude toward risk, he might hedge using any number of contracts between 80,000 and 100,000.

However, the most common way of thinking is to say that farmer Brown wants to address what happens on average, i.e., to hedge the expected number of bushels. In this problem we get

\[ \frac{1}{4} \times 100000 + \frac{3}{4} \times 80000 = 25000 + 60000 = 85000. \]

1.2. **Payoff and profit curves for a user/buyer of goods who hedges using a forward contract.**

**Problem 1.4.** Pancakes, Inc. produces chocolate chip pancakes. It longed a forward contract on 100 lbs of chocolate chips at \$3.00 per pound. Total fixed revenue is \$2,000 for the pancakes produced with the above chocolate chips. Other costs total \$1200. Find the company’s profit.

(a) 2,000
(b) 1,700
(c) 800
(d) 500
(e) None of the above.
Solution: (d)

\[ 2000 - 1200 - 300 = 500 \]
Problem 1.5. The market price of the good is the independent argument $s$. Assume that the purchaser is hedging by longing a forward contract. Draw the graph of the purchaser’s profit as a function of $s$.

Solution: See our lecture notes.

Problem 1.6. You are a jeweler who buys silver – the primary input needed for your products. Exactly one ounce of silver is used to produce one unit of jewelry. Assume that the cost of all other inputs is negligible. You are able to sell each unit of jewelry for 700 plus 20% of the market price of silver in one year. In one year, the actual price of silver is modeled as being in one of three possible states, corresponding to the following probability table:

<table>
<thead>
<tr>
<th>Market Price of Silver in 1 year</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = 250$ per ounce</td>
<td>$p_1 = 0.3$</td>
</tr>
<tr>
<td>$x_2 = 300$ per ounce</td>
<td>$p_2 = 0.4$</td>
</tr>
<tr>
<td>$x_3 = 350$ per ounce</td>
<td>$p_3 = 0.3$</td>
</tr>
</tbody>
</table>

You are considering utilizing forward contracts to lock in 1-year silver prices, in which case you would charge the customer (one year from now) 700 plus 20% of the forward price. The forward price of silver for delivery in one year is 300 per ounce. How much does your expected 1-year profit, per unit of jewelry sold, increase if you buy the silver forward?

Solution:

Problem 1.7. The Extra-Healty Cereal (EHC) company longed 20,000 forward contracts on corn at $2.80 per bushel. The revenue from cereal made with the above corn is $200,000 while the other (non-corn) aggregate fixed and variable costs amount to $120,000. What is the EHC’s profit?

Solution:

$$200000 - 120000 - 20000 \times 2.80 = 24,000.$$ 

Assume that the corporate tax rate equals 35%. What is EHC’s after-tax profit?

Solution:

$$24,000 \times 0.65 = 15,600.$$ 

1.3. Forward contracts.

Problem 1.8. The time-0 spot price of the market index is $900. A 3-month forward contract on this market index has the forward price of $930. After 3 months the market index is priced at $920. The nominal rate of interest convertible monthly is 4.8%. What is the difference in the profits between a long index investment and a long forward contract investment?

Solution: The difference between the payoffs is just the forward price of $930. Since there is no initial cost in a forward contract, the initial cost of the entire position consist merely of the investment in the index. It equals $900.

So, the difference between the profits is

$$920 - (920 - 930) - 900(1.004)^3 = 19.1567.$$