University of Texas at Austin

Problem set 3

Continuous-dividend-paying stocks.

Problem 3.1. You have \$300 to invest in a market index worth \$100 per unit. The market index pays dividends continuously with the dividend yield equal to 0.02. How many units of the market index will you own in six months?

Solution: Let n_0 denote the initial number of units of the market index you are able to purchase at time-0. Then, $n_0 = 3$.

Let $N(t)$ stand for the number of units of the market index you own at time$-t$ (with the convention of continuous and immediate reinvestment of dividends in the same asset). Then,

$$N(1/2) = 3e^{(0.02)(0.5)} = 3e^{0.01} \approx 3.03015.$$

Problem 3.2. Consider a certain stock which pays dividends continuously with the dividend yield of 0.03. How many shares would you need to purchase today to ensure that you own **exactly** one share in a quarter year? Moreover, let today's price of this stock be \$80 per share. How much does your investment cost you?

Solution: Let the number of shares you need to buy today be denoted by n_0. Then, with $N(t)$ denoting the number of shares you own at time$-t$, we have the following condition:

$$n_0 e^{0.03(0.25)} = 1 \quad \Rightarrow \quad n_0 = e^{-0.0075} \approx 0.992528.$$

The initial cost would be $80e^{-0.0075} \approx 79.4022$.