Forward contracts. European call options. European put options.

2.1. Calls.

Problem 2.1. The initial price of a non-dividend-paying asset is $100. A six-month, $95-strike European call option is available at a $8 premium.

The continuously compounded risk-free interest rate equals 0.04.

What is the break-even point for this call option?

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

Solution: (d)

We need to solve for $s$ in

\[(s - 95)_+ = 8e^{0.02} \Rightarrow s = 95 + 8e^{0.02} = 103.16\]

Problem 2.2. (5 points) Which of the following constitutes a one-year, $100-strike covered call?

(a) Write a one-year, $100-strike call and short the underlying asset.
(b) Write a one-year, $100-strike call and buy the underlying asset.
(c) Buy a one-year, $100-strike call and short the underlying asset.
(d) Buy a one-year, $100-strike call and buy the underlying asset.
(e) None of the above.

Solution: (b)

Problem 2.3. (5 points) A stock’s price today is $1000 and the annual effective interest rate is given to be 5%. You write a one-year, $1,050-strike call option for a premium of $10 while you simulataneously buy the stock. What is your profit if the stock’s spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $10.50
(d) - $39.00
(e) None of the above.

Solution: (c)

\[S(T) - 1000(1.05) - (S(T) - K)_+ + 10(1.05) = 1050 - 990(1.05) = 10.50.\]

Problem 2.4. (5 points) The price of gold in half a year is modeled to be equally likely to be any of the following prices $1000, $1100, and $1240.

Consider a half-year, $1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

(a) $63.33
(b) $80.00
(c) $125.00

Solution: (b)
(d) $1113.33
(e) None of the above.

Solution: (b)

$$50 \times \frac{1}{3} + 200 \times \frac{1}{3} = \frac{240}{3} = 80.$$ 

Problem 2.5. (20 points)
The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will able to sell every piece for $1,000.

The jeweler models the market price of gold in one year as follows:

<table>
<thead>
<tr>
<th>Gold price in one year</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>750 per ounce</td>
<td>0.2</td>
</tr>
<tr>
<td>850 per ounce</td>
<td>0.5</td>
</tr>
<tr>
<td>950 per ounce</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The jeweler hedges the price of gold by buying a 1−year call option with an exercise price of $900 per ounce. The option costs $100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the hedged portfolio per piece of jewelery produced.

Solution:

With $S(T)$ denoting the market price of gold at time $T = 1$, the jeweler’s hedged profit per piece of jewelry can be expressed as

$$1,000 - \min(S(T), 900) - 100e^{0.05} = 894.873 - \min(S(T), 900).$$

So, her expected hedged profit equals

$$894.873 - \mathbb{E}[\min(S(T), 900)] = 894.873 - (0.2 \cdot 750 + 0.5 \cdot 850 + 0.3 \cdot 900) = 49.873.$$
2.2. Puts.

Problem 2.6. The initial price of the market index is $900. After 3 months the market index is priced at $915. The nominal rate of interest convertible monthly is 4.8%. The premium on the put, with a strike price of $930, is $8.00. What is the profit at expiration for a long put?

(a) $15.00 loss
(b) $6.90 loss
(c) $6.90 gain
(d) $15.00 gain
(e) None of the above.

Solution: (c)
The profit from a position is defined as the position’s payoff minus the future value of the initial cost.

If \( S(T) = 915 \) denotes the price of the market index at time \( T = 0.25 \) (i.e., in three months), then the payoff of the long put is \((K - S(T)+)\), where \( K = 930 \) denotes the strike of the put. So, since \( K > S(T) \), the payoff is

\[(930 - 915) = 15.\]

The future value of the initial put premium is \( 8(1 + 0.004)^3 = 8.0964 \).

So, the profit is

\[15 - 8.0964 = 6.90.\]

Problem 2.7. Sample FM(DM) #12
Consider a European put option on a stock index without dividends, with 6 months to expiration, and a strike price of 1,000. Suppose that the nominal annual risk-free rate is 4% convertible semiannually, and that the put costs 74.20 now. What price must the index be in 6 months so that being long the put would produce the same profit as being short the put?

A. 922.83
B. 924.32
C. 1,000.00
D. 1,075.68
E. 1,077.17

Solution: (b)
Method I. A quick and insightful way of solving this problem is by realizing that the long-put and the short-put profits are negatives of each other. So, the only way they can be equal is at the “break-even” point. We solve for \( s \) in

\[(K - s) + V_P(0) \left(1 + \frac{i^{(2)}}{2}\right) = (1000 - s) + 74.20(1.02) = 0.\]

The solution is \( s = 924.32 \).

Method II. This is the more pedestrian method. The long-put profit is

\[(K - s) + V_P(0) \left(1 + \frac{i^{(2)}}{2}\right) = (1000 - s) + 74.20(1.02).\]

The short-put profit is the exact negative of the expression above, i.e.,

\[-(K - s) + V_P(0) \left(1 + \frac{i^{(2)}}{2}\right) = -(1000 - s) + 74.20(1.02).\]

So, algebraically, we need to solve for \( s \) in the equation

\[(1000 - s) + 74.20(1.02) = -(1000 - s) + 74.20(1.02) \iff 2(1000 - s) + 2 \cdot 74.20(1.02) \iff (1000 - s) = 74.20(1.02).\]
We get the same answer as above, of course.

**Problem 2.8.** Farmer Shaun is producing sweet potatoes. He intends to harvest 10,000—cartons’ worth in six months. His total costs are $12.00 per carton.

He wishes to hedge using European put options. There are two puts on sweet potatoes with the exercise date in six months available: one with the strike price of $13 per carton and another with the strike price of $15 per carton. Their premiums are $0.15 and $0.18, respectively.

Assume that the prevailing risk-free interest rate is 4% effective for the half-year period.

At harvest time, in six months, it turns out that the sweet-potato spot price equals $14. What would Farmer Shaun’s profit be if he had decided to hedge using the $13-strike put versus his profit if he had decided to use the $15-strike put to hedge?

**Solution:** Farmer Shaun’s unhedged position has the following profit:

\[ 10,000(S(T) - 12) \]

where \( S(T) \) stands for the spot price of sweet potatoes in six months.

If he decided to hedge using put options, he would **long** the put. So, the profit of the $13-strike-put hedge would be:

\[ 10,000(13 - S(T))_+ - 10,000 \times 0.15 \times 1.04. \]

The profit of the $15-strike-put hedge would be:

\[ 10,000(15 - S(T))_+ - 10,000 \times 0.18 \times 1.04. \]

The profit of the hedged position with the given \( S(T) = 14 \) in the first case equals

\[ 10,000(14 - 12 - 0.15 \times 1.04) = 18,440. \]

For the second insurance strategy, the profit is

\[ 10,000(14 - 12 + (15 - 14) - 0.18 \times 1.04) = 28,128. \]

2.3. **Forwards.**

**Problem 2.9.** Roger owns a cow named Elsie. Her estimated worth today is $3,750. Roger enters into a forward agreement with Harry to sell him Elsie the cow in 6 months for $4,000. On the delivery date, Roger changes his mind and wants cash settlement instead. Harry agrees. They look into the “Bovine Blue Book” and realize that Elsie’s worth on that date is $3,500.

What is the cash flow that has to take place as part of the cash settlement?

(a) $500 from Roger to Harry
(b) $500 from Harry to Roger
(c) $250 from Roger to Harry
(d) $250 from Harry to Roger
(e) None of the above.

**Solution:** (b)

**Problem 2.10.** (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

(a) an arbitrageur.
(b) a broker.
(c) a speculator.
(d) a hedger.
(e) None of the above.

**Solution:** (d)