Please, provide complete solution(s) to the following problem(s):

**Problem 5.1.** (15 points) The current price of stock is observed to be 80 per share. Its dividend yield is 0.02 and its volatility is 0.3.

The continuously compounded risk-free interest rate equals 0.03.

We model the stock price at the end of the next quarter using a forward binomial tree.

(i) (5 points) What is the risk-neutral probability of the stock price going up consistent with the above forward binomial tree?

(ii) (10 points) What is the price of a (75, 90)-collar on the above stock with exercise date in a quarter year?

**Solution:** The risk-neutral probability is

\[ p^* = \frac{1}{1 + e^{\sigma \sqrt{h}}} = \frac{1}{1 + e^{0.15}} = 0.46257 \]

The up and down factors are

\[ u = e^{(\delta h + \sigma \sqrt{h})} = e^{0.0025 + 0.15} = 1.16474, \]
\[ d = e^{(\delta h - \sigma \sqrt{h})} = e^{0.0025 - 0.15} = 0.862862. \]

Therefore, the two possible stock prices are

\[ S_u = 93.1792, \quad S_d = 69.029. \]

The payoff of the ratio spread is

\[ V(h) = (K_P - S(h))_+ - (S(h) - K_C)_+ \]

So, the two possible payoff values are

\[ V_u = -3.1792, \quad V_d = 5.971. \]

The time–0 price of our ratio spread is, hence,

\[ V(0) = e^{-0.03(0.25)}[ -3.1792 \times 0.46257 + 5.971 \times (1 - 0.46257)] = 1.7254. \]