Problem 6.1. (2 points) You are using a binomial asset-pricing model to model the evolution of the price of a particular stock. Then, it is possible for the ∆ in the replicating portfolio of a call option on that stock to exceed 1.

Solution: FALSE
The call’s ∆ will always be between 0 and 1.

Problem 6.2. (2 points) Source: Problem 10.12 from the McDonald text.
Let \( S(0) = 40, r = 0.08, \sigma = 0.3, \delta = 0 \). You need to construct a 2-period forward binomial tree with each period on length one year for the above stock. Then, \( u > 1.31 \).

Solution: TRUE

\[
u = \exp\{(0.08 - 0) \cdot 1 + 0.3\sqrt{1}\} \approx 1.46.
\]

Problem 6.3. (2 points) It is never optimal to exercise an American call option on a non-dividend-paying stock early. True or false?

Problem 6.4. (2 points)
American-style options are at least as valuable as otherwise identical European-style options. True or false?

Problem 6.5. (2 points) An Asian arithmetic-average-strike call option is at least as valuable as an otherwise identical Asian geometric-average-strike option. True or false?

Problem 6.6. (5 points) Let \( A(T) \) denote the arithmetic average of a set of observed stock prices, and let \( G(T) \) denote the geometric average of the same set of observed stock prices. Which one of the following inequalities is always correct?

(a) \( (K - A(T))_+ \geq (K - G(T))_+ \)
(b) \( (A(T) - K)_+ \geq (G(T) - K)_+ \)
(c) \( (A(T) - K)_+ \geq (S(T) - K)_+ \)
(d) \( (S(T) - K)_+ \geq (G(T) - K)_+ \)
(e) None of the above.

Solution: (b) We already established that \( A(T) \geq G(T) \). The equality holds iff all of the stock-prices entering the calculation are equal. This implies that (a) is not always correct while (b) is always correct.

To rule out (c) and (d), consider the simple situation in which just two stock-prices are sampled: \( S(T/2) \) and \( S(T) \). In case that \( S(T/2) < S(T) \), we get

\[
A(T) = \frac{1}{2}(S(T/2) + S(T)) < S(T) \quad \Rightarrow \quad (S(T) - K)_+ > (A(T) - K)_+.
\]

In case that \( S(T/2) > S(T) \), we get

\[
G(T) = \sqrt{S(T/2)S(T)} > S(T) \quad \Rightarrow \quad (G(T) - K)_+ > (S(T) - K)_+.
\]