Problem 3.1. (5 points)
Let $x_L \leq x^* \leq x_R$. Express $x^*$ as a convex combination of $x_L$ and $x_R$. More precisely, find $\lambda$ such that

$$x^* = \lambda x_L + (1 - \lambda) x_R.$$ 

Definition 3.1. A function $f : [0, \infty) \to \mathbb{R}$ is said to be convex if for every $x_L < x_R$ and every $0 \leq \lambda \leq 1$ we have

$$f(\lambda x_L + (1 - \lambda) x_R) \leq \lambda f(x_L) + (1 - \lambda) f(x_R)$$

Problem 3.2. (5 points)
Provide an example of a convex function $f : [0, \infty) \to \mathbb{R}$ (by drawing its graph in a coordinate system or by providing the explicit expression for the function).

Problem 3.3. (10 points)
The increasing function $f : [0, \infty) \to \mathbb{R}$ satisfies the following inequality for every choice of $x_1 < x_2 < x_3$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} \leq \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

Show that the function $f$ is convex.