Problem 13.1. (2 points)

Early exercise of an American call option only makes sense when there is a positive cashflow on the underlying asset prior to the expiration date of the option. True or false?

Solution: TRUE

Problem 13.2. Consider a non-dividend paying asset whose initial price is $100 per share. The evolution of the stock price over the following 9-month interval is modeled by a three-period binomial tree with the up factor \( u = 2 \) and the down factor \( d = u^{-1} \).

The effective interest rate per quarter-year is given to be equal to 0.25.

i. (5 points) What is the risk-neutral probability of the stock price going up in a single period?

ii. (10 points) What is the price of a 9-month, at-the-money European call option on the above stock?

iii. (20 points) What is the price of a 9-month, $50-strike American put option on the above stock?

Solution:

i. 

\[ p^* = \frac{1 + i - d}{u - d} = \frac{1.25 - 0.5}{2 - 0.5} = 0.5 \]

ii. 

\[ V_C(0) = \left(\frac{4}{5}\right)^3 \left[ (800 - 100) \times \left(\frac{1}{2}\right)^3 + 3(200 - 100) \times \left(\frac{1}{2}\right)^3 \right] = 64 \]

iii. The only non-zero payoff should the option be held onto until the expiration date is at the \( ddd \) node. We have \( V_{ddd} = 50 - 12.5 = 37.5 \). So, the continuation value at the \( dd \) node is

\[ CV_{dd} = \left(\frac{4}{5}\right) \times \frac{1}{2} \times 37.5 = 75 \times \frac{1}{5} = 15. \]

On the other hand, the value of immediate exercise at the same node equals \( IE_{dd} = 50 - 25 = 25 \). We conclude that it is optimal to early exercise at the \( dd \) node and that the value of our American put at that node equals \( V_{dd}^A = 25 \). At the \( d \) node, the option is at the money, so it does not make sense to exercise it early. Similarly, the option is not optimally exercised early at the \( ROOT \) node. So,

\[ V_P^A(0) = \left(\frac{4}{5}\right)^2 \times \frac{1}{4} \times 25 = 4. \]