Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.
Time: 50 minutes

## TRUE/FALSE

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## MULTIPLE CHOICE

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1.1. DEFINITIONS.

Problem 1.1. (10 points) Write the definition of an arbitrage portfolio.

Solution:
See the homework solutions.

1.2. TRUE/FALSE QUESTIONS. Please, circle the correct answer on the front page of this exam.

Problem 1.2. A covered call consists of a written call and long underlying asset.

Solution: TRUE

1.3. FREE-RESPONSE PROBLEMS.

Problem 1.3. (20 points) Suppose that the current price of a dividend-paying stock equals $100. Let \( r = 0.05 \) and \( \delta = 0.03 \). You notice that a forward price for delivery of this stock in two-years equals \( F = $110 \). You suspect that this forward price creates an arbitrage opportunity.

Construct an arbitrage portfolio which consists of taking a position in the forward contract, purchase or short-sale of the underlying asset, and borrowing/lending at the risk-free interest rate.

Solution: The forward price based on the initial stock price, \( r \) and \( \delta \) equals

\[
F_{0,T}(S) = S(0)e^{(r-\delta)T} = 100e^{(0.05-0.03)\cdot 2} \approx 104.08 < F = 110.
\]

The conclusion is that the observed forward price is “too high”. One way to exploit this arbitrage opportunity would be to do the following:

1. engage in the short forward contract,
2. borrow \( S(0)e^{-\delta T} \) at the risk free rate to be repaid at time \( T \),
3. use the above loan to buy \( e^{-\delta T} \) shares of stock.

So, the initial cost of this portfolio is zero.

During the time period \((0,T]\), all of the continuously paid dividends are automatically reinvested in the asset \( S \). So, at the end, one share of stock is owned. If we want to, we can deliver it to fulfill the forward contract. Thus, at time–\( T \), the payoff is

\[
-(S(T) - F) + S(T) - e^{(r-\delta)T}S(0) > 0.
\]

The portfolio we constructed is, indeed, an arbitrage portfolio.
1.4. **MULTIPLE CHOICE QUESTIONS.** Please, circle the correct answer on the front page of this exam.

**Problem 1.4.** (5 points) The premium on a 2-month call option on the market index with an exercise price of 1050 is $9.30 when originally purchased. After 2 months the position is closed and the index spot price is 1072. If interest rates are 0.5% effective per month, what is the call’s profit?

(a) $9.30  
(b) $9.39  
(c) $12.61  
(d) $22.00  
(e) None of the above.

**Solution:** (c)

The value at expiration of the cost of the call is $9.30 \times 1.005^2 \approx 9.39.

The payoff of the call is 1072 − 1050 = 22.

So the profit is 22 − 9.39 = 12.61.

**Problem 1.5.** (5 points) The spot price of the market index is $900. After 3 months the market index is priced at $940.

An investor has a long call option on the index at a strike price of $930. After 3 months what is the investors payoff?

(a) $10 loss  
(b) $0  
(c) $10 gain  
(d) $20 gain  
(e) None of the above.

**Solution:** (c)

In our usual notation, the payoff is

\[(S(T) - K)_+ = (940 - 930)_+ = 10.\]

**Problem 1.6.** The initial price of the market index is $900. After 3 months the market index is priced at $960. The effective monthly rate of interest is 1.0%.

The premium on the long put, with a strike price of $975, is $10.00. What is the profit at expiration for this long put?

(a) $2.00 loss  
(b) $4.70 loss  
(c) $4.70 gain  
(d) $2.00 gain  
(e) None of the above.
Solution: (c)
The profit is

\[(K - S(T))_+ - FV_{0,T}[V_P(0)] = (K - S(T))_+ - FV_{0,T}[V_P(0)]
\]
\[= (975 - 960)_+ - 10(1 + 0.01)^3
\]
\[= 4.70.
\]

Problem 1.7. The current price of the market index is $900. The continuously compounded risk-free interest rate is 4.8%. After 3 months the market index is priced at $920. What is the profit for the writer of the $930-strike, three-month call option if the time-0 option premium equals $2.00?

(a) About $12.02 loss
(b) About $2.02 loss
(c) About $2.02 gain
(d) About $12.02 gain
(e) None of the above.

Solution: The option is not exercised, so the profit is

\[2e^{0.048/4} = 2.02
\]

Problem 1.8. (5 points) A non-dividend-paying stock sells for $100 per share today. The one-year forward price is $110. You short sell the stock and close the short sale in exactly one year. Find your profit if the stock’s spot price in one year equals $130 per share.

(a) 20 loss
(b) 20 gain
(c) 30 loss
(d) 30 gain
(e) None of the above.

Solution: (a)
Because the stock pays no dividends, we have that \(FV_{0,T}(S(0)) = F_{0,T}(S)\). So, the profit equals

\[-S(1) - FV_{0,1}(-S(0)) = -S(1) + F_{0,1}(S) = -130 + 110 = -20.
\]
Problem 1.9. A market index is currently trading at $1,000. Which of the following options is/are in the money? More than one answer can be true. You get the credit if you circled all acceptable answers and none of the incorrect ones.

(a) $1,500-strike put
(b) $900-strike put
(c) $1,250 strike call
(d) $950 strike call
(e) None of the above.

Solution: (a) and (d)