Notes: This is a closed book and closed notes exam. The maximal score on this exam is 100 points.

Time: 50 minutes

MULTIPLE CHOICE

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TRUE/FALSE

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FOR THE GRADER’S USE ONLY:

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1.1. DEFINITIONS.

Problem 1.1. (10 points) Write the definition of an arbitrage portfolio.

1.2. TRUE/FALSE QUESTIONS. Please, circle the correct answer on the front page of this exam.

Problem 1.2. (2 pts) A (long) put is a short position with respect to the underlying asset price.

Solution: TRUE

Problem 1.3. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date $T$ on the underlying asset $S$:
- long one call with strike 40,
- long two calls with strike 50,
- short one call with strike 65.

Let $S(T) = 69$. Then, the payoff from the above position at time $T$ is less than 60.

Solution: FALSE

The payoff is

$$(69 - 40) + 2(69 - 50) - (69 - 65) = 63.$$ 

Problem 1.4. It is possible for the buyer and the writer of the same option to end up having the same profit on the exercise date.

Solution: TRUE

This happens if they both break even, i.e., if both of their profits equal zero.
1.3. **FREE-RESPONSE PROBLEMS.**

Please, record your final numerical answers in the boxes at the bottom of every page with a free-response problem. Single-word or one-number answers without a justification will receive zero credit.

**Problem 1.5.** (20 points) A certain continuous-dividend-paying stock is currently priced at $100. Its dividend yield is given to be 0.01.

The continuously compounded risk-free interest rate is given to be 0.04.

You observe that a forward contract for delivery of the above stock in 2 years for the forward price of $F = 110$ is available in the market.

- Is there an arbitrage opportunity available? If your answer is affirmative, substantiate your suspicion. If your answer is negative, show why.
- If your answer was positive, propose an arbitrage portfolio.
- Verify that your proposed arbitrage portfolio is, indeed, an arbitrage portfolio.

**Solution:**

*Diagnosis.* The only source of arbitrage is the observed forward price. So, let us compare it to the no-arbitrage forward price as dictated by the given data

$$F_{0,T}(S) = S(0)e^{(r-\delta)T} = 100e^{2\times(0.04-0.01)} = 100e^{0.06} \approx 106.18 < 110 = F.$$  

Since the no-arbitrage forward price is strictly smaller than the observed forward price, there is, indeed, an arbitrage opportunity.

*Arbitrage portfolio.* Since the observed forward price is greater than the no-arbitrage forward price, we conclude that the observed forward contract is relatively expensive. So, we decide to **short** it. To counteract this decision, we need to **long** an appropriate number of shares of stock sufficient to fulfill the obligations of the short forward contract. In summary, we propose the following arbitrage portfolio:

- **short** the observed forward contract;
- **purchase** $e^{-\delta T}$ shares of stock.

*Verification.* The **initial cost** of the above portfolio is $S(0)e^{-\delta T}$. The **payoff** of the portfolio is

$$F - S(T) + e^{\delta T}e^{\delta T}S(T) = F.$$

Finally, the **profit** equals

$$F - e^{\delta T}e^{-\delta T}S(0) = F - F_{0,T}(S) > 0.$$  

Hence, our portfolio is, indeed, an arbitrage portfolio.
Problem 1.6. (20 points)
The primary ingredient for a certain jeweler is gold which she intends to buy in exactly one year. She considers all of her other production-related expenses to be negligible.

The jeweler uses exactly one ounce of gold to produce every one of her pieces, and will be able to sell every piece for $1,000.

The jeweler models the market price of gold in one year as follows:

<table>
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<th>Gold price in one year</th>
<th>Probability</th>
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<tr>
<td>750 per ounce</td>
<td>0.2</td>
</tr>
<tr>
<td>850 per ounce</td>
<td>0.5</td>
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<tr>
<td>950 per ounce</td>
<td>0.3</td>
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The jeweler hedges the price of gold by buying a 1-year call option with an exercise price of $900 per ounce. The option costs $100 per ounce now.

The continuously compounded risk-free interest rate is 5%.

Calculate the expected profit of the hedged portfolio per piece of jewelry produced.

Solution:
With \( S(T) \) denoting the market price of gold at time \( T = 1 \), the jeweler’s hedged profit per piece of jewelry can be expressed as

\[
1,000 - \min(S(T), 900) - 100e^{0.05} = 894.873 - \min(S(T), 900).
\]

So, her expected hedged profit equals

\[
894.873 - E[\min(S(T), 900)] = 894.873 - (0.2 \cdot 750 + 0.5 \cdot 850 + 0.3 \cdot 900) = 49.873.
\]
1.4. **MULTIPLE CHOICE QUESTIONS.**

*Please, circle the correct answer on the front page of this exam.*

**Problem 1.7.** (5 points) Which of the following constitutes a one-year, $100-strike covered call?

(a) Write a one-year, $100-strike call and short the underlying asset.
(b) Write a one-year, $100-strike call and buy the underlying asset.
(c) Buy a one-year, $100-strike call and short the underlying asset.
(d) Buy a one-year, $100-strike call and buy the underlying asset.
(e) None of the above.

**Solution:** (b)

**Problem 1.8.** (5 points) A stock’s price today is $1000 and the annual effective interest rate is given to be 5%. You write a one-year, $1,050-strike call option for a premium of $10 while you simultaneously buy the stock. What is your profit if the stock’s spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $10.50
(d) - $39.00
(e) None of the above.

**Solution:** (c)

\[
S(T) - 1000(1.05) - (S(T) - K)_{+} + 10(1.05) = 1050 - 990(1.05) = 10.50.
\]

**Problem 1.9.** (5 points) The price of gold in half a year is modeled to be equally likely to be any of the following prices $1000, $1100, and $1240.

Consider a half-year, $1050-strike European call option on gold. What is the expected payoff of this option according to the above model?

(a) $63.33
(b) $80.00
(c) $125.00
(d) $1113.33
(e) None of the above.

**Solution:** (b)

\[
50 \times \frac{1}{3} + 190 \times \frac{1}{3} = \frac{240}{3} = 80.
\]
Problem 1.10. (5 points) A stock now sells for $100 per share. The continuously compounded risk-free interest rate equals 0.05, and the continuous dividend yield equals 0.02. Find the prepaid forward price for delivery in two years.

(a) 96.08  
(b) 97.04  
(c) 98.02  
(d) 100  
(e) None of the above.

Solution: (a)

\[ F^P_{0.2}(S) = S(0)e^{-2\delta} = 100e^{-0.04} = 96.08. \]

Problem 1.11. (5 points) A soy-bean farmer shorts forward contracts on soy in an amount matching his crop volume and with delivery at harvest time. Then, he is considered:

(a) an arbitrageur.  
(b) a broker.  
(c) a speculator.  
(d) a hedger.  
(e) None of the above.

Solution: (d)

Problem 1.12. The payoff of an asset-or-nothing call option with strike price \( K \) and exercise date \( T \) is given by:

(a) \( (S(T) - K)_+ \)  
(b) \( 1_{[S(T) \geq K]} \)  
(c) \( S(T)1_{[S(T) \geq K]} \)  
(d) \( (S(T) - K)1_{[S(T) \geq K]} \)  
(e) None of the above.

Solution: (e)
Problem 1.13. The initial price of a non-dividend-paying asset is $100. A six-month, $95-strike European call option is available at a $8 premium. The continuously compounded risk-free interest rate equals 0.04. What is the break-even point for this call option?

(a) 86.84
(b) 87
(c) 103
(d) 103.16
(e) None of the above.

Solution: (d)
We need to solve for $s$ in

$$(s - 95)_+ = 8e^{0.02} \Rightarrow s = 95 + 8e^{0.02} = 103.16$$

Problem 1.14. (5 points) A non-dividend-paying stock sells for $100 per share today. The one-year forward price is $110. You short sell the stock and close the short sale in exactly one year. Find your profit if the stock’s spot price in one year equals $130 per share.

(a) 20 loss
(b) 20 gain
(c) 30 loss
(d) 30 gain
(e) None of the above.

Solution: (a)
Because the stock pays no dividends, we have that $FV_{0,T}(S(0)) = F_{0,T}(S)$. So, the profit equals

$$-S(1) - FV_{0,1}(-S(0)) = -S(1) + F_{0,1}(S) = -130 + 110 = -20.$$

Problem 1.15. A market index is currently trading at $1,000. Which of the following options is/are in the money? More than one answer can be true. You get the credit if you circled all acceptable answers and none of the incorrect ones.

(a) $1,500-strike put
(b) $900-strike put
(c) $1,250 strike call
(d) $950 strike call
(e) None of the above.

Solution: (a) and (d)