Continuous-dividend-paying stocks

$S(t)$... stock price at time $t$ for $t \geq 0$

*Stochastic process*

\[ \begin{align*}
0 & \quad t & \quad T
\end{align*} \]

$\delta$... dividend yield

The shareholders receive $SS(t) dt$ during the time interval $(t, t+dt)$ per share owned.

Q: What is the total amount of dividend paid during $[0, T]$?

\[ \begin{align*}
\text{Discrete} & \quad \sum_{k=1}^{T} D_k \\
\text{Continuous} & \quad \int_{0}^{T} SS(t) dt
\end{align*} \]

Convention: With continuous dividends, all dividend payments are IMMEDIATELY & CONTINUOUSLY REINVESTED in the same asset.
The amount paid per share owned is \( S \cdot S(t) \, dt \).

\[ N(t) \text{ ... number of shares owned at time } t \]

\[ N(0) = n_0 \]

\[ \frac{dN(t)}{dt} = ? \text{ rate of change of } N \]

\[ \frac{dN(t)}{dt} = N(t+dt) - N(t) \]

\[ \Rightarrow dN(t) = S N(t) \, dt \]

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\[ N(0) = n_0 \]

\[ \Rightarrow N(t) = n_0 e^{\delta t} \]

\[ \text{The amount of dividend during } (t, t+dt) \text{ is:} \]

\[ N(t) S S(t) \, dt \]

\[ \Rightarrow \text{The } N(t) S S(t) \, dt \text{ is } \frac{N(t) S S(t) \, dt}{S(t)} \]
Example. Invest $200 in a market index worth $50 per unit today. The dividend yield is 0.03. How many units do you own one year from now?

\[ n_0 = 4 \]

\[ N(1) = \frac{4 \cdot e^{0.03 \cdot 1}}{4} = \ldots \]

Example. Dividend yield for an index equals 0.05. How many units do you have to buy today in order to ensure that you own exactly 1 unit half a year from now?

\[ N(\frac{1}{2}) = 1 = n_0 \cdot e^{0.05 \cdot \frac{1}{2}} \]

\[ \Rightarrow n_0 = e^{-0.025} \]
Example. Invest $1,000 in a market index currently worth $100 per unit. The dividend yield is 0.02.

How many units do you own 2 years from now?

\[
\rightarrow: \quad N(2) = 10e^{0.02 \cdot 2} = 10e^{0.04} = \ldots \]

Example. Dividend yield for an index equals 0.01. How many units do you have to buy today in order to ensure that you own exactly 1 unit half a year from now?

\[
\rightarrow: \quad 1 = N(\frac{1}{2}) = n_0 e^{0.01 \cdot \frac{1}{2}}
\]

\[
\Rightarrow \quad n_0 = e^{-0.005}
\]
Static Portfolios

No Intermediate Cashflows

The Payoff:

The time-T Cashflow from the investor's perspective.

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The time-T Cashflow from the investor's perspective.

Example 1: Harry purchases a zero-coupon bond for $1,000 with a continuously compounded yield of 0.05 and maturity at time-2.

\[ \text{Initial cost} : \$1,000 \]
\[ \text{Payoff} : \$1,000 e^{0.05 \cdot 2} = 1,000 e^{0.1} \]

Example 2: Harry takes out a simple five-year loan of $1,000. The loan is to be repaid in full at time-5. The continuously compounded interest charged is 0.04.

\[ \text{Initial Cost} : \ -\$1,000 \]
\[ \text{Payoff} : \ -\$1,000 e^{0.04 \cdot 5} = -1,000 e^{0.2} \]
Example 3: Outright Purchase of 1 share of non-dividend-paying stock.

Diagram:
- Stock price $S(0)$ vs. time $T$
- $S(T)$... final market price of the stock