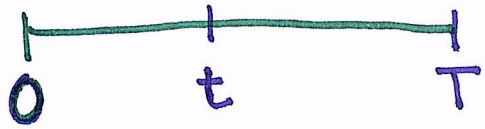
09/04/15

Continuous-dividend-paying stocks

$S(t)$... stock price @ time-t for $t \geq 0$

* Stochastic process *



δ.... dividend yield

The shareholders receive $\delta S(t)dt$ during the time interval $(t, t+dt)$ per share owned.

Q: What is the total amount of dividend paid during $[0, T]$?

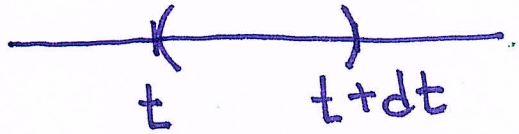
Discrete

$$\sum_{k=1}^{n} D_k$$

Continuous

$$\boxed{\int_0^T \delta S(t)\,dt}$$

$\boxed{\text{Convention:}}$ With continuous dividends, all dividend pmts are IMMEDIATELY & CONTINUOUSLY REINVESTED in the same asset.



Amt paid is $\delta \cdot S(t)\,dt$ per share owned.

$N(t)$ ... number of shares owned @ time-t

$$\boxed{N(0) = n_0}$$

$$\frac{dN(t)}{dt} = ?$$ rate of change of N

$N(t)$ $\quad$ $N(t+dt)$

$$dN(t) = N(t+dt) - N(t)$$

The amt of dividend during $(t, t+dt)$ is:

$$\boxed{N(t)\delta S(t)dt}$$

$\Rightarrow$ The # of shares bought during $(t, t+dt)$ is $\dfrac{N(t)\delta \cancel{S(t)}dt}{\cancel{S(t)}}$

$$\Rightarrow \quad dN(t) = \delta N(t)dt$$

$$\left.\begin{aligned} &\Rightarrow \quad \frac{dN(t)}{dt} = \delta N(t) \\ &N(0) = n_0 \end{aligned}\right\} \Rightarrow \boxed{N(t) = n_0 e^{\delta t}}$$

Example. Invest \$200 in a market index worth \$50 per unit today.
The dividend yield is 0.03.
How many units do you own one year from now?

$\rightarrow$: $n_0 = 4$

$$N(1) = \underbrace{4e^{0.03 \cdot 1}} = \dots$$

Example. Dividend yield for an index equals 0.05.
How many units do you have to buy today in order to ensure that you own exactly 1 unit half a year from now?

$$N(1/2) = 1 = n_0 e^{0.05 \cdot \frac{1}{2}}$$

$$\Rightarrow \boxed{n_0 = e^{-0.025}}$$

<u>Example</u>. Invest \$1,000 in a market index currently worth \$100 per unit.
The dividend yield is 0.02.
How many units do you own 2 years from now?

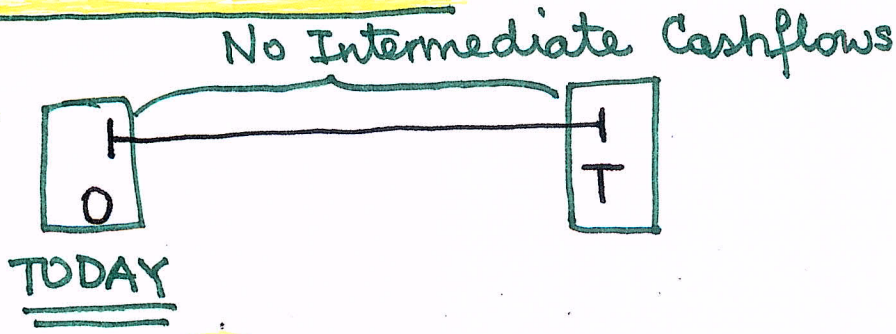
$$\rightarrow: \quad N(2) = 10e^{0.02\cdot 2} = \underbrace{10e^{0.04}} = \dots \quad \blacksquare$$

<u>Example</u>. Dividend yield for an index equals 0.01.
How many units do you have to buy today in order to ensure that you own exactly 1 unit half a year from now?

$$\rightarrow: \quad 1 = N(1/2) = n_0 e^{0.01\cdot\frac{1}{2}}$$

$$\Rightarrow \quad n_0 = e^{-0.005} \quad \blacksquare$$

Static Portfolios



Initial Cost

The time-0 CASHFLOW from the investor's perspective.

The Payoff:

The time-T CASHFLOW from the investor's perspective.

Example 1. Harry purchases a zero-coupon bond for \$1,000 w/ a continuously compounded yield of 0.05 and maturity @ time-2.

$\Rightarrow$ Initial cost: \$1,000

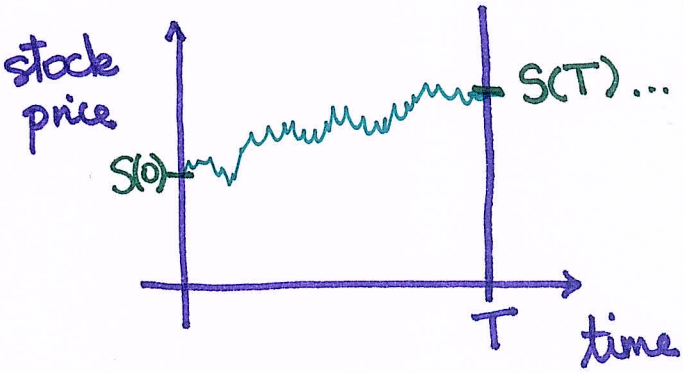
PAYOFF: $\$1{,}000 \cdot e^{0.05 \cdot 2} = 1{,}000 e^{0.1}$

Example 2. Harry takes out a simple five-year loan of \$1,000. The loan is to be repaid in full @ time-5. The continuously compounded interest charged is 0.04.

$\Rightarrow$ Init. Cost: $-\$1{,}000$

PAYOFF: $-\$1{,}000 e^{0.04 \cdot 5} = -1{,}000 e^{0.2}$

Example 3: Outright Purchase of 1 share of non-dividend-paying stock.



S(T)... final market price of the stock