**Notes:** This is a closed book and closed notes exam. The maximal score on this exam is 100 points.

**Time:** 75 minutes

### TRUE/FALSE

| 1 (2) | TRUE | FALSE |
| 2 (5) | a b c d e |
| 3 (5) | a b c d e |
| 4 (5) | a b c d e |
| 5 (5) | a b c d e |
| 6 (5) | a b c d e |
| 7 (5) | a b c d e |
| 8 (5) | a b c d e |

### MULTIPLE CHOICE

| 1 (5) | a b c d e |
| 2 (5) | a b c d e |
| 3 (5) | a b c d e |
| 4 (5) | a b c d e |
| 5 (5) | a b c d e |
| 6 (5) | a b c d e |
| 7 (5) | a b c d e |
| 8 (5) | a b c d e |
TRUE/FALSE QUESTIONS

Please, circle the correct answer on the front page of this exam.

1. (2 pts) Consider a portfolio consisting of the following four European options with the same expiration date $T$ on the underlying asset $S$:
   - long one call with strike 40,
   - long two calls with strike 50,
   - short one call with strike 65.
   Let $S(T) = 69$. Then, the payoff from the above position at time $T$ is less than 60.
   Solution: FALSE
   The payoff is
   $$(69 - 40) + 2(69 - 50) - (69 - 65) = 63.$$  

2. (2 pts) Source: Sample FM(DM) Problem #27.
   The position consisting of one long homeowners insurance contract benefits from falling prices in the underlying asset.
   Solution: TRUE
   Recall our comparison of the homeowner’s insurance policy to the put option. The payoff of the put option is decreasing in the price of the underlying asset.

FREE-RESPONSE PROBLEMS

1. (10 points) The fair price today of a zero-coupon bond with redemption amount of $100 and which comes to maturity in a year is equal to $78.
   You purchase an at-the-money European call option on a non-dividend paying stock whose price today is $S(0) = 100$. The premium of this call was $10$.
   Write the expression for this call’s payoff, and for its profit (valued at its expiration date $T$) as a function of $S(T)$ (the stock price at time $T$) and the time of maturity $T$.
   Draw the graph of this calls profit as a function of $S(T)$.
   Solution: From the bond price, denoting by $r$ the annual continuously compounded interest rate, we get $e^r = 100/78$.
   So, the expression for the call’s profit is
   $$(S(T) - 100)^+ - 10e^{rT} = (S(T) - 100)^+ - 10\left(\frac{100}{78}\right)^T.$$  
   Here is what the profit function looks like for $T = 1$. For other maturities, you get the same shape.
2. (20 points) Which of the positions listed will benefit from the underlying asset’s price decline? Draw the payoff curves for each position and justify your answer.

(i) Short put
(ii) Long put
(iii) Short call
(iv) Short stock
(v) Short forward contract

Solution: Only the short put is long in the underlying asset.

3. (10 points) A strategy consists of buying a market index product at $830 and longing a put on the index with a strike of $830. The put premium is $18.00 and interest rates are 0.5% effective per month. Compute the profit or loss from this position in 6 months if the market index is worth $810 at time 6 months.

Solution: The profit from a position is defined as the position’s payoff minus the future value of the initial cost.

If \( S(T) \) denotes the price of the market index at time \( T = 0.5 \) (i.e., in six months), then the payoff of the long position in the index is exactly \( S(T) \). The payoff of the long put is \( (K - S(T))_+ \), where \( K = 830 \) denotes the strike of the put. So, since \( K > S(T) \), the total payoff of this position is

\[
S(T) + (K - S(T))_+ = S(T) + K - S(T) = K = 830.
\]

The initial cost of the above position is \( S(0) + V_P(0) \) where \( S(0) \) denotes the initial index price and \( V_P(0) \) denotes the put premium. So, the future value of the total initial cost is

\[
(830 + 18)(1.005)^6 = 873.76.
\]

The final answer is, thus, \( 830 - 873.76 = -43.76 \).
4. (10 points) For what values of the final asset price is the profit of a long forward contract with the forward price $F = 100$ and delivery date $T$ in one year smaller than the profit of a long call on the same underlying asset with the strike price $K = 100$ and the exercise date $T$. Assume that the call’s premium equals $10$ and that the annual effective interest rate equals $10\%$.

Express your answer as an interval.

**Solution:** The profit function of the forward contract is $v_F(s) = s - 100$. The profit function of the call is

$$v_C(s) - 10 \times 1.10 = (s - 100)_+ - 11.$$

For $s \geq 100$, the call’s profit is smaller than the forward contract’s profit. So, we focus on $s < 100$. Here we have to solve for $s^*$ in

$$s^* - 100 = -11 \Rightarrow s^* = 89.$$

The answer is $[0, 89)$.

**MULTIPLE CHOICE QUESTIONS**

Please, circle the correct answer on the front page of this exam.

1. (5 points) The spot price of the market index is $900$. After 3 months the market index is priced at $940$. An investor has a long call option on the index at a strike price of $930$. After 3 months what is the investor’s payoff?
   - (a) $10$ loss
   - (b) $0$
   - (c) $10$ gain
   - (d) $20$ gain
   - (e) None of the above.

**Solution:** (c)

In our usual notation, the payoff is

$$(S(T) - K)_+ = (940 - 930)_+ = 10.$$

2. (5 points) If your homeowner’s insurance premium is $1,000$ and your deductible is $2,000$, what could be considered the strike price of the policy if the home has a value of $120,000$?
   - (a) $117,000$
   - (b) $118,000$
   - (c) $120,000$
   - (d) $122,000$
   - (e) None of the above.
Solution: (b)
The analogue of the strike price for this homeowner’s insurance policy is the home’s initial value minus the deductible, i.e., \(120,000 - 2,000 = 118,000\).

3. (5 points) Farmer Jayne bought a $1.70-strike put option for $0.11 and sold a $1.75-strike call option for a premium of $0.14. Both options expire in six months. Her total costs of producing the corn are $1.65 per bushel. She will sell the 20,000-bushel corn crop in six months. Assume that the effective interest rates for a six month period are 4.0% What is the minimum profit in her strategy?
(a) $624
(b) $1,624
(c) $2,624
(d) $3,624
(e) None of the above.
Solution: (b)
Per bushel, Farmer Jayne’s profit is
\[
(K_1 - S(T))_+ - (S(T) - K_2)_+ - 1.65 + S(T) - (V_P(0) - V_C(0))(1 + i)
\]
where \(S_T\) denotes the price of a bushel of corn in six months, i.e., at time \(T = 0.5\), and where \(K_1 = 1.70, K_2 = 1.75, V_C(0) = 0.14, V_P(0) = 0.11\) and \(i = 0.04\). If you draw the graph of the profit above, you will notice that it is nondecreasing in \(S_T\). This means that the minimum profit occurs for \(S_T = 0\). In that case, the profit for the 20,000 bushels of corn is
\[
20,000(1.70 - 1.65 - (-0.03)(1.04)) = 1,624.
\]

4. The initial price of the market index is $900. After 3 months the market index is priced at $920. The nominal rate of interest convertible monthly is 4.8%.
The premium on the long call, with a strike price of $930, is $2.00. What is the profit at expiration for this long call?
(a) $2.00 loss
(b) $2.02 loss
(c) $2.02 gain
(d) $2.00 gain
(e) None of the above.
Solution: (b)
In our usual notation, the profit is
\[
(S(T) - K)_+ - C \cdot (1 + j)^3
\]
with \(C\) denoting the price of the call and \(j\) the effective monthly interest rate. We get
\[
(920 - 930)_+ - 2 \cdot 1.04^3 \approx -2.02.
\]
5. **Source: Prof. Jim Daniel (personal communication).**
The bid-ask spread on a share of stock is $98-$102. A 5% commission is paid for either buying or selling. Calculate the round-trip transaction cost.

(a) $14
(b) $10
(c) $6
(d) $4
(e) None of the above.

**Solution:** (a)
You spend $102 \times (1 + 0.05) = 107.10$ to buy the asset, and receive $98 \times (1 - 0.05) = 93.10$ when you sell the asset. The round-trip transaction cost is $107.10 - 93.10 = 14.$

6. **Source: Prof. Jim Daniel (personal communication).**
A stock’s price today is $1000 and the annual effective interest rate is given to be 10%. You write a one-year, $1,050-strike call option for a premium of $10 while you simultaneously buy the stock. What is your profit if the stock’s spot price in one year equals $1,200?

(a) $150.00
(b) $139.90
(c) $60.00
(d) - $39.00
(e) None of the above.

**Solution:** (d)
\[
S(T) - 1000(1.10) - (S(T) - K)_+ + 10(1.10) = 1050 - 1010(1.10) = -39.
\]
7. Roger owns a cow named Elsie. Her estimated worth today is $3,750. Roger enters into a forward agreement with Harry to sell him Elsie the cow in 6 months for $4,000. On the delivery date, Roger changes his mind and wants cash settlement instead. Harry agrees. They look into the “Bovine Blue Book” and realize that Elsie’s worth on that date is $3,500.

What is the cash flow that has to take place as part of the cash settlement?
(a) $500 from Roger to Harry
(b) $500 from Harry to Roger
(c) $250 from Roger to Harry
(d) $250 from Harry to Roger
(e) None of the above.

Solution: (b)

8. A market index is currently trading at $1,000. Which of the following options is/are in the money? More than one answer can be true. You get the credit if you circled all acceptable answers.
(a) $1,500-strike put
(b) $900-strike put
(c) $1,250 strike call
(d) $950 strike call
(e) None of the above.

Solution: (a) and (d)

SHORT ANSWER QUESTIONS

1. (10 points) From a manufacturer’s perspective, why would he/she decide to use derivative securities on their product to hedge? Respond in five lines or less.

Solution: Answers may vary, but the bottom line is that the manufacturer can prevent losses only by a limited amount using operations optimisation and other tools within his/her area of expertise. Their profit still depends heavily on market-price fluctuations – well outside of his/her area of influence and/or expertise. So, derivative securities are a welcome tool to hedge that risk.

2. (10 points) Believe it or not, we all enter forward contracts on a daily basis. Explain, in six lines or fewer, how ordering a pizza is a forward contract. Ignore time-limits on when the pizza should be delivered. Imagine that the pizza is to be delivered in 30 minutes exactly.

Solution: The quantity and type of pizza is specified at order time. So is the agreed upon price and the delivery time. The logistics are also explained (you give your address and contact info). The pizza parlor will confirm that you, indeed, ordered your pizza. While there are no formal steps to alleviate “credit risk” (you decide you do not want the pizza, even though you did order it, and you refuse to pay), you might get “blacklisted” by a particular pizza parlor.