SAMPLE MFE

7. Company A is a U.S. international company, and Company B is a Japanese local company. Company A is negotiating with Company B to sell its operation in Tokyo to Company B. The deal will be settled in Japanese yen. To avoid a loss at the time when the deal is closed due to a sudden devaluation of yen relative to dollar, Company A has decided to buy at-the-money dollar-denominated yen put of the European type to hedge this risk.

You are given the following information:

(i) The deal will be closed 3 months from now. \[ T = \frac{1}{4} \]

(ii) The sale price of the Tokyo operation has been settled at 120 billion Japanese yen. \[ 120 \cdot 10^9 \]

(iii) The continuously compounded risk-free interest rate in the U.S. is 3.5%. \[ r_g = r_D = 0.035 \]

(iv) The continuously compounded risk-free interest rate in Japan is 1.5%. \[ r_F = 0.015 \]

(v) The current exchange rate is 1 U.S. dollar = 120 Japanese yen. \[ x(0) = \frac{1}{120} \]

(vi) The natural logarithm of the yen per dollar exchange rate is an arithmetic Brownian motion with daily volatility 0.261712%. \[ \sigma = \frac{0.261712}{100} \times \sqrt{365} \]

(vii) 1 year = 365 days; 3 months = \( \frac{1}{4} \) year.

\[ \sigma = 0.05 \]

Calculate Company A's option cost.

\[(vi) \Rightarrow \{ x^{-1}(t), t \geq 0 \} \text{ is a GBM with volatility } \sigma = \frac{0.261712}{100} \times \sqrt{365} \]

\[ \sigma = 0.05 \]

The underlying asset for our put is \( x \) (not \( x^{-1} \)). If \( x^{-1} \) is a GBM, then \( x \) is also a GBM.

\[ \Rightarrow \text{Black-Scholes pricing works.} \]
\[ V_p(0) = Ke^{-\frac{d_2^T}{2}}N(-d_2) - x(o)e^{-tF^{-1}}N(-d_1) \]

\[ d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{x(o)}{K} \right) + \left( \frac{F_{\sigma T}^p(x)}{2} - \frac{\sigma^2}{2} \right) T \right] ; \]

\[ d_2 = d_1 - \sigma \sqrt{T} . \]

- \( K = x(o) \) since @ the money
- \( \sigma = 0.05 \) (\( x \) and \( x^{-1} \) have the same 1x1 as we showed already!)

**In this problem:**

\[ d_1 = \frac{1}{0.05 \sqrt{0.4}} \left[ \ln \left( \frac{1}{1} \right) + \left( 0.035 - 0.015 + \frac{(0.05)^2}{2} \right) \cdot \frac{1}{4} \right] \]

\[ = \frac{20}{\sqrt{2}} \left( 0.02 + \frac{0.0025}{2} \right) \cdot \frac{1}{4} = 10 \left( 0.02 + 0.00125 \right) \]

\[ d_1 = 0.2125 ; \quad d_2 = 0.2125 - 0.05 \cdot \frac{1}{2} = 0.1875 \]

\[ \Rightarrow N(-d_1) = 1 - N(d_1) = 0.4168 \]

\[ N(-d_2) = 1 - N(d_2) = 0.4286 \]

\[ \Rightarrow V_p(0) = \frac{1}{120} \left( e^{-0.035 \cdot \frac{1}{4}} \cdot 0.4286 - e^{-0.015 \cdot \frac{1}{4}} \cdot 0.4168 \right) \]

**answer:** \( 120 \cdot 10^9 \cdot \frac{1}{120} \left[ \begin{array}{c}
9.6 \\
10^9
\end{array} \right] = 9.6 \cdot 10^9 \)

**Compare this answer to the "official" one!**
Futures options

\[ \delta \quad \text{dividend yield} \quad \leftrightarrow \quad r \quad \text{interest rate} \quad \text{futures} \]

cont-div-stocks

For a call on a futures contract:

\[ V_C(0) = e^{-rT} \mathcal{F}_{0,T_F} \cdot N(d_1) - e^{rT} K \cdot N(d_2) \]

\[ V_C(0) = e^{-rT} \left[ \mathcal{F}_{0,T_F} \cdot N(d_1) - K \cdot N(d_2) \right] \]

with

\[ d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{\mathcal{F}_{0,T_F}}{K} \right) + (r - r^2 + \frac{\sigma^2}{2}) \cdot T \right] \]

\[ d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{\mathcal{F}_{0,T_F}}{K} \right) + \frac{\sigma^2 \cdot T}{2} \right] \]

and

\[ d_2 = d_1 - \sigma \sqrt{T} \]
55. Assume the Black-Scholes framework. Consider a 9-month at-the-money European put option on a futures contract. You are given:

(i) The continuously compounded risk-free interest rate is 10%.

(ii) The strike price of the option is 20.

(iii) The price of the put option is 1.625.

If three months later the futures price is 17.7, what is the price of the put option at that time?

\[ V_p(0) = e^{-rT}K\left[N(-d_2) - N(-d_1)\right] \]

\[ V_p(0) = e^{-rT}K\left[N\left(\frac{\sigma\sqrt{T}}{2}\right) - N\left(-\frac{\sigma\sqrt{T}}{2}\right)\right] \]

\[ = e^{-rT}K\cdot2N\left(\frac{\sigma\sqrt{T}}{2}\right) - 1 \]

In this problem:

\[ 1.625 = e^{-0.10\cdot0.25}\cdot20\cdot\left[2\cdot N\left(\frac{\sigma\sqrt{0.75}}{2}\right) - 1\right] \]

\[ N\left(\frac{\sigma\sqrt{3}}{4}\right) = 0.54 \quad \Rightarrow \quad \sigma \approx 0.25 \]
At time $\frac{1}{4}$:

\[ V_p \left( \frac{1}{4} \right) = e^{-rT} \left[ 20 \cdot N(-d_2^{\text{new}}) - 17.7 \cdot N(-d_1^{\text{new}}) \right] \]

with \[ d_1^{\text{new}} = \frac{1}{0.25 \cdot \sqrt{\frac{1}{2}}} \left[ \ln \left( \frac{17.7}{20} \right) + \frac{(0.25)^2}{2} \cdot \frac{1}{2} \right] \approx -0.59 \]

\[ \Rightarrow d_2^{\text{new}} = -0.77 \]

\[ \Rightarrow V_p \left( \frac{1}{4} \right) = 2.66 \Rightarrow \bigcirc \]