More on Market-Making and Delta-Hedging
What do market makers do to delta-hedge?

- Recall that the delta-hedging strategy consists of **selling** one option, and **buying** a certain number $\Delta$ shares.

- An example of Delta hedging for 2 days (daily rebalancing and mark-to-market):

**Day 0:** Share price = $40, call price is $2.7804, and $\Delta = 0.5824$
- Sell call written on 100 shares for $278.04, and buy 58.24 shares.
- Net investment: $(58.24 \times 40) - 278.04 = 2051.56$
- At 8%, overnight financing charge is $0.45 = 2051.56 \times (e^{-0.08/365} - 1)$

**Day 1:** If share price = $40.5, call price is $3.0621, and $\Delta = 0.6142$
- Overnight profit/loss: $29.12 - 28.17 - 0.45 = 0.50$ (mark-to-market)
- Buy 3.18 additional shares for $128.79 to rebalance

**Day 2:** If share price = $39.25, call price is $2.3282
- Overnight profit/loss: $76.78 + $73.39 - 0.48 = 3.87$ (mark-to-market)
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Self-Financing Trading: Discrete Time

- Let $X_k$ denote the value of the hedging portfolio at time $k$
- Let $\Delta_k$ denote the number of shares of stock held between times $k$ and $k + 1$
- At time $k$, after rebalancing (i.e., moving from a position of $\Delta_{k-1}$ to a position of $\Delta_k$), the amount we hold in the money market account is

$$X_k - S_k \Delta_k$$

- The value of the portfolio at time $k + 1$ is

$$X_{k+1} = \Delta_k S_{k+1} + (1 + r)(X_k - \Delta_k S_k)$$
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Self-Financing Trading: Discrete Time - The Gain

- So, the gain between time $k$ and time $k + 1$ is

$$X_{k+1} - X_k = \Delta_k (S_{k+1} - S_k) + r (X_k - \Delta_k S_k)$$

- This means that the gain is the sum of the capital gain from the stock holdings:

$$\Delta_k (S_{k+1} - S_k)$$

and the interest earnings from the money-market account

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Define the value of a share in the money-market account at time $k$ to be

$$M_k = (1 + r)^k$$

and let the number of shares of the money-market held at time $k$ be denoted by $\Gamma_k$
Self-Financing Trading: Discrete Time
- The new expression for the gain

- So, the gain between time $k$ and time $k + 1$ can now be written as

$$X_{k+1} - X_k = \Delta_k (S_{k+1} - S_k) + \Gamma_k (M_{k+1} - M_k)$$

- Thus, the gain is the sum of the capital gain from the stock investment holdings:

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- The wealth at time $k + 1$ can be expressed as

$$X_{k+1} = \Delta_k S_{k+1} + \Gamma_k M_{k+1}$$

- However, $\Delta_k$ and $\Gamma_k$ cannot be chosen arbitrarily.
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Self-Financing Trading: Discrete Time
- The self-financing condition

- The agent arrives at time $k + 1$ with a portfolio of $\Delta_k$ shares of stock and $\Gamma_k$ shares of the money market account and then rebalances, i.e., chooses $\Delta_{k+1}$ and $\Gamma_{k+1}$
- After rebalancing, the wealth of the agent is
  \[ X_{k+1} = \Delta_{k+1} S_{k+1} + \Gamma_{k+1} M_{k+1} \]
- The wealth of the agent cannot be changed through rebalancing, so it must be that
  \[ X_{k+1} = \Delta_{k+1} S_{k+1} + \Gamma_{k+1} M_{k+1} = \Delta_k S_{k+1} + \Gamma_k M_{k+1} \]
- The last equality yields the discrete time self-financing condition
  \[ S_{k+1}(\Delta_{k+1} - \Delta_k) + M_{k+1}(\Gamma_{k+1} - \Gamma_k) = 0 \]
- The first term is the cost of rebalancing the stock and the second term is the cost of rebalancing the money-market account
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Self-Financing Trading:
Segue into the continuous time

- The discrete-time self-financing condition can be rewritten as

\[ S_k(\Delta_{k+1} - \Delta_k) + (S_{k+1} - S_k)(\Delta_{k+1} - \Delta_k) \]
\[ + M_k(\Gamma_{k+1} - \Gamma_k) + (M_{k+1} - M_k)(\Gamma_{k+1} - \Gamma_k) = 0 \]

- This suggests the continuous-time self-financing condition

\[ S_t \, d\Delta_t + dS_t \, d\Delta_t + M_t \, d\Gamma_t + dM_t \, d\Gamma_t = 0 \]

- This claim can be proved using stochastic calculus ....
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Recall the meaning of Delta

- An option written on an underlying asset $S$ is most sensitive to the changes in the value of $S$
- The largest part of the risk comes from the price movements of asset $S$ - which is reflected in the delta of the option, i.e., if $C$ is the price of a call the most pronounced effect comes from

$$\Delta C := \frac{\partial C}{\partial S}$$

- The replicating portfolio will always contain $\Delta C$ shares of the underlying stock
- The portfolio which contains the option, along with $\Delta C$ shares of stock will have the value of its Delta equal to zero - we say it is delta neutral
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Recall the other Greeks

- If $X$ denotes the price of a portfolio, we define

*Theta:* 
\[ \Theta := \frac{\partial X}{\partial t} \]

*Gamma:* 
\[ \Gamma := \frac{\partial^2 X}{\partial S^2} \]

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$$ \Gamma := \frac{\partial^2 X}{\partial S^2} $$

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The Delta-Gamma-Theta Approximation

- In this model \( X \) depends only on the values of \( S \) and \( t \) (\( \sigma \) and \( r \) are assumed constant)
- The Taylor expansion of \( X \) gives us
  \[
  X(t + \Delta t, S + \Delta S) = X(t, s) + \frac{\partial X(t, S)}{\partial S} \Delta S + \frac{\partial X(t, S)}{\partial t} \Delta t + \frac{1}{2} \frac{\partial^2 X(t, S)}{\partial S^2} (\Delta S)^2 + \text{higher order terms}
  \]
  where \( \Delta S = S(t + \Delta t) - S(t) \)
- Ignoring the higher order terms (as one would in establishing the Ito’s Lemma), we get
  \[
  X(t + \Delta t, S + \Delta S) \approx X(t, s) + \Delta \cdot \Delta S + \Theta \cdot \Delta t + \frac{1}{2} \Gamma \cdot (\Delta S)^2
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- So, we can aim to reduce the variability of the portfolio by making \( \Delta \) and \( \Gamma \) small - we cannot do much about \( \Theta \).
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Understanding the Market-Maker’s Portfolio

- Suppose that, at any time $t \leq T$, the market-maker is long $\Delta(t)$ shares of stock and short one call on that stock.
- Then the cost/profit of his/her portfolio at time $t$ equals

$$Y(t, S_t) = \Delta(t) \cdot S_t - C(t, S_t)$$

where $C(t, S_t)$ is the price of the call at time $t$.
- Suppose that the cost/profit above is invested in the money-market account.
- Then, the change in the value of the portfolio is

$$\Delta(t) \cdot \Delta S - (C(t + \Delta t, S(t + \Delta t)) - C(t, S(t))) - r \cdot \Delta t \cdot Y(t, S_t)$$
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The Taylor approximation similar to the one we conducted earlier yields that the change in the market-maker’s portfolio is approximately

$$- \left( \frac{1}{2} (\Delta S)^2 \cdot \Gamma + \Delta t \cdot \Theta + r \Delta t \cdot (\Delta \cdot S_t - C(t, S(t))) \right)$$

where the Greeks are calculated at time $t$.
The Merton-Black-Scholes model

- In the Black-Scholes setting, for an option with price $C = C(t, s)$ we have that

$$\left(\Delta S\right)^2 \approx \left(dS\right)^2 = \sigma^2 S^2 \, dt$$

- So, we get

$$\Theta + \frac{1}{2} \sigma^2 S^2 \Gamma + r(S\Delta - C) = 0$$

- Note that the value of $\Theta$ is fixed once we fix the values of $\Delta$ and $\Gamma$
- Note that $\Gamma$ is the measure of risk the hedger faces as a result of not rebalancing frequently enough
- so, in the absence of transaction costs, the agent should rebalance often as this reduces the variance of his/her portfolio
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- The recipe for Delta-hedging is the same as for the European option, but restricted to the continuation region, i.e., the region prior to early exercise (if it is to happen).
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- As we have seen thus far, the market-maker may adopt a **Delta-neutral** position to try to make her portfolio less sensitive to uncertainty, i.e., the changes in $S$

- Alternatively, one may adopt a **Gamma-neutral** position by using options to hedge - it is necessary to use other types of options for this strategy

- Augment the portfolio by by buying deep-out-of-the-money options as insurance - this is probably not a viable strategy

- Use **static option replication** according to put-call parity to form a both Delta- and Gamma-neutral hedge

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Gamma-Neutrality

- Denote by $\Gamma$ the gamma of a certain portfolio $X$ and by $\Gamma_C$ the gamma of a certain contingent claim $C$.
- In addition to having $X$, we want to buy/sell $n$ contracts of $C$ in order to make the entire portfolio gamma-neutral, i.e., we want to have
  \[ \Gamma + n\Gamma_C = 0 \]
- So, the correct number of contingent claims $C$ to buy/sell is
  \[ n = -\frac{\Gamma}{\Gamma_C} \]
- However, this addition to our position changes the delta of the entire portfolio.
- To rectify this, we trade a certain (appropriate) number of the shares of the underlying asset to make the entire portfolio delta-neutral, as well.
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- Consider a Delta-neutral portfolio \( X \) that has \( \Gamma = -5,000 \)
- Assume that the traded option has \( \Delta_C = 0.4 \) and \( \Gamma_C = 2 \)
- We offset the negative gamma of the portfolio \( X \) by purchasing \( n = 5,000/2 - 2,500 \) option contracts
- The resulting portfolio is gamma-neutral, but has the delta equal to

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\Delta_{new} = 2,500\Delta_C = 2,500 \cdot 0.4 = 1,000
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- To rectify this, we should sell 1,000 shares of the underlying asset
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  1. Hold capital reserves
  2. Diversify risk by buying reinsurance

- Market-makers also have two analogous ways to deal with excessive losses:
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