2.1. **Black-Derman-Toy.** Provide the **final answer** to the following problem(s):

**Problem 2.1.** (5 points) The price of a zero-coupon bond redeemable in one year equals $93.50, while the price of a zero-coupon bond redeemable in two years equals $82.50.

You are using the above bond prices to calibrate a Black-Derman-Toy tree of effective annual interest rates under the additional assumption that the volatility of interest rates in the second period equals 0.09.

Let \( r_d \) denote the interest rate in the “down” state. Then, \( r_d \) falls within the following interval:

(a) \([0, 0.07)\)
(b) \([0.07, 0.09)\)
(c) \([0.09, 0.12)\)
(d) \([0.12, 0.14)\)
(e) None of the above.
Provide a complete solution to the following problem(s):

**Problem 2.2.** (8 points) **Source:** Problems 24.12 and 24.13 from the textbook.
Here is an incomplete Black-Derman-Toy interest rate tree with effective annual interest rates at each node.

(i) (2 pts) Calculate $r_{ud}$.
(ii) (3 pts) What is the 3-year zero coupon bond price per $100 at maturity implied by this tree? Assume that the bond is issued at time 0.
(iii) (3 pts) What volatilities of annual effective interest rates were used to construct the above tree?
**Problem 2.3.** (8 points) *Source: Problems 24.12 and 24.13 from the McDonald textbook.*

Here is an incomplete Black-Derman-Toy interest rate tree with effective annual interest rates at each node.

\[
\begin{array}{c}
0.08000 \\
0.08112 \\
0.10689 \\
0.09908 \\
0.08749 \\
0.10689
\end{array}
\]

(i) (2 pts) Calculate \( r_{uu} \).

(ii) (3 pts) What is the 3-year zero coupon bond price per $100 at maturity implied by this tree? Assume that the bond is issued at time 0.

(iii) (3 pts) What volatilities of annual effective interest rates were used to construct the above tree?

**Problem 2.4.** (10 points)

Consider the following values of interest rates from an incomplete Black-Derman-Toy interest rate tree for the effective annual interest rates.

\[
\begin{align*}
    r_0 &= 0.10, & r_u &= 0.15, & r_{uu} &= 0.19, \\
    r_d &= 0.13, & r_{ud} &= 0.15
\end{align*}
\]

Calculate the yield volatility in year one of a zero-coupon bond with maturity three years from today.

**Problem 2.5.** (10 points) A three-period Black-Derman-Toy tree is calibrated so that

\[
\begin{align*}
    r_0 &= 0.03, & r_u &= 0.04, & r_{uu} &= 0.05, \\
    r_d &= 0.035, & r_{ud} &= 0.04.
\end{align*}
\]

Consider a one-year, $0.93-strike put option on a zero-coupon bond which matures at time \(-3\) for $1. What is the price of this put option consistent with the above interest-rate model?

**Problem 2.6.** (10 points) The evolution of short-term interest rates is modeled using the Black-Derman-toy tree with base rate parameters

\[
\begin{align*}
    r_0 &= 0.06, & r_d &= 0.05, & r_{dd} &= 0.045, & r_{ddd} &= 0.04,
\end{align*}
\]

and volatilities of effective interest rates

\[
\sigma_1 = 0.10, \sigma_2 = 0.12, \sigma_3 = 0.15.
\]

Consider a four-year caplet with cap rate of 0.08 on a $1000 interest-only loan with end-of-year payment installments. What is today’s price of this caplet?

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**Instructor:** Milica Čudina