Problem 2.1. (10 points) A continuous-dividend-paying stock is valued at $75.00 per share. Its dividend yield is 0.03. The time−t realized (rate of) return is modeled as

\[ R(0, t) \sim N(\text{mean} = 0.035t, \text{variance} = 0.09t) \]

Find the probability that the time−4 stock price exceeds today’s stock price.

Solution: We need to find \( P[S(4) > S(0)] \)

with

\[ S(t) = S(0)e^{R(0,t)}. \]

Since \( R(0,t) \) follows the normal distribution with the above parameters, we have

\[ P[S(4) > S(0)] = P[S(0)e^{R(0,4)} > S(0)] = P[R(0,4) > 0] = 1 - N\left(-\frac{0.035 \times 4}{0.3 \times 2}\right) = N(0.23) = 0.591. \]

Problem 2.2. (10 points) A continuous-dividend-paying stock is valued at $75.00 per share. Its dividend yield is 0.03. The time−t realized (rate of) return is modeled as

\[ R(0, t) \sim N(\text{mean} = 0.035t, \text{variance} = 0.09t) \]

An investor purchases a single share of stock at time−0 and continuously (and immediately) reinvests any dividends received in the same asset. What are the mean and median values of the investor’s position at time−4?

Solution: The expected rate of return (per annum) is

\[ 0.035 + 0.03 + \frac{1}{2} \times 0.09 = 0.035 + 0.03 + 0.045 = 0.11. \]

The mean is

\[ e^{0.45}E[S(T)] = 75e^{0.11} = 116.45. \]

Similarly, the median is

\[ 116.45 \times e^{-0.09 \times 4/2} = 97.27. \]

Problem 2.3. (10 points) A non-dividend-paying stock is valued at $75.00 per share. The annual expected (rate of) return is 16.0% and the standard deviation of annualized returns is given to be 0.30. If the stock price is modeled using the lognormal distribution (as discussed in class), what is the constant \( s_{1/2}^U \) such that \( P[S(1/2) > s_{1/2}^U] \leq 0.05 \).

Solution: Note that the 95th percentile of the standard normal distribution equals 1.645. So,

\[ s_{1/2}^U = 75e^{(0.16 - \frac{1}{2} \times 0.3^2) \times \frac{1}{2} + 0.3 \times \frac{1}{2} \times 1.645} = 112.61. \]

Problem 2.4. (5 points) Source: Problem 18.10 in McDonald.

Let the stock price at time 1 be denoted by \( S(1) \) and modeled using the lognormal distribution.

You are given that, in the usual notation, \( S(0) = 100, \alpha = 0.08, \sigma = 0.3 \) and \( \delta = 0 \). Find \( P[S(1) < 98] \).
Solution: We have

\[ P(S(1) < 98) = N(-\hat{d}_2). \]

with \(-\hat{d}_2 = (\ln(98/100) - 0.035)/0.3 = -0.18401\). Hence,

\[ P(S(1) < 98) = N(-0.18401) = 0.427. \]

Problem 2.5. (5 pts) For a stock price that was initially $55.00, what is the price after 4 years if the continuously compounded returns for these 4 years are 4.5%, 6.2%, 8.9%, and −3.2%?

Solution:

\[ 55e^{0.045+0.062+0.089-0.032} \approx 64.80. \]

Problem 2.6. (5 pts) A stock is valued at $55.00. The annual expected return is 12.0% and the standard deviation of annualized returns is 22.0%. If the stock is lognormally distributed, what is the expected stock price after 3 years?

Solution: Let us denote the stock price today by \( S(0) \) and that in three years by \( S(3) \). According to the work we did in class, we need to calculate

\[ E[S(3)] = S(0)e^{3\alpha} \]

with \( \alpha \) equal to the expected continuously compounded rate of return on the stock \( S \). We are given in the problem that \( \alpha = 0.12 \). So, the answer is \( 55e^{0.36} \approx 78.83. \)

Problem 2.7. (5 pts) For a stock price that was initially $55.00, what is the price after 4 years if the observed continuously compounded returns for these 4 years are 4.5%, 6.2%, 8.9%, and 3.2%?

Solution:

\[ 55e^{0.045+0.062+0.089+0.032} \approx 69.08. \]