University of Texas at Austin

HW Assignment 4


Provide your final answer only to the following problem(s):

**Problem 4.1.** (2 points)
Let \( \{X_n, n = 1, 2, \ldots \} \) be a sequence of independent, identically distributed continuous random variables with the density \( f_{X_n} \). Assume that the second moment of \( X_1 \) is well defined.

According to the Law of Large Numbers,
\[
\frac{X_1^2 + X_2^2 + \cdots + X_n^2}{n} \to \int_{-\infty}^{\infty} x^2 f_X(x) \, dx
\]

**Problem 4.2.** (5 points) Solve Sample MFE Problem #33.

**Problem 4.3.** (5 points) There are two stocks present in our market: \( S \) and \( Q \). Their current prices are \( S(0) = 60 \) and \( Q(0) = 65 \). Both stocks pay dividends continuously. The dividend yield for \( S \) is 0.02 while the dividend yield for \( Q \) equals 0.03.

You are given that for \( t \geq 0 \)
\[
\text{Var}[\ln(S(t)/Q(t))] = 0.04t.
\]

What is the Black-Scholes price of a one-year exchange call with underlying \( S \) and the strike asset \( Q \)?

(a) $2.86
(b) $3.01
(c) $7.27
(d) $7.86
(e) None of the above.

**Problem 4.4.** (5 points) The price of a zero-coupon bond redeemable at time– for $1 is denoted by \( P(0, T) \).
You are given the following bond prices
\[
P(0, 1) = 0.93, \quad P(0, 2) = 0.86, \quad P(0, 3) = 0.77, \quad P(0, 4) = 0.70.
\]

The volatilities of the forward prices for delivery at time–k of the above bonds at one year to maturity equal are denoted by \( \sigma_k \). We are given
\[
\sigma_1 = 0.10, \quad \sigma_2 = 0.11, \quad \sigma_3 = 0.12.
\]

What is the Black price of a $0.92-strike, two-year European call option on the bond with one year to maturity on the exercise date?

(a) $0.0968
(b) $0.058
(c) $0.0368
(d) $0.028
(e) None of the above.

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**Problem 4.5.** (5 points) The price of a zero-coupon bond redeemable at time $T$ for $\$1$ is denoted by $P(0, T)$. You are given the following bond prices

$$P(0, 1) = 0.92, \quad P(0, 2) = 0.85, \quad P(0, 3) = 0.76, \quad P(0, 4) = 0.70.$$ 

The volatilities of the forward prices for delivery at time $-k$ of the above bonds at one year to maturity equal $\sigma_k$. We are given

$$\sigma_1 = 0.10, \quad \sigma_2 = 0.12, \quad \sigma_3 = 0.14.$$ 

What is the Black price of a $0.92$-strike, two-year European call option on the bond with one year to maturity on the exercise date?

(a) $\$0.013$

(b) $\$0.023$

(c) $\$0.033$

(d) $\$0.0419$

(e) None of the above.

**Problem 4.6.** (5 points) In our usual notation,

$$P(0, 1) = 0.95, \quad P(0, 2) = 0.88, \quad P(0, 3) = 0.80.$$ 

The volatility of a two-year forward on a zero-coupon bond with one year left to maturity is $0.10$. What is the Black price of a two-year, $0.95$-strike put option on a bond with one year left to maturity?

Provide a complete solution to the following problem(s):

**Problem 4.7.** (18 points) Source: Problem #14.18 from McDonald (2nd Ed).

Consider two assets with prices recorded as processes $S$ and $Q$ and an exchange call with $S$ as the price of the underlying asset and $Q$ as the price of the strike asset. The call's maturity is $T = 1$ and the risk-free interest rate is $r = 0.08$.

Let $S(0) = 40, \sigma_S = 0.3$ and $\delta_S = 0$ and let $Q(0) = 60, \sigma_Q = 0.5$ and $\delta_Q = 0$. The correlation between the two prices is $\rho = 1$.

i. (6 pts) Find the price of the above described call.

ii. (6 pts) Change the volatility of the asset $Q$ so that $\sigma_Q = 0.4$ Find the new price of the above described call.

iii. (4 pts) Find the connection between $S$ and $Q$ in both of the above cases, i.e., find a formula connecting the two prices.

iv. (2 pts) Using your answer to part iii., explain the effect on the price that you noticed in comparing the answers to part i. and part ii.

**Problem 4.8.** (10 points) You are given the following prices of zero-coupon bonds, in our usual notation,

$$P(0, 1) = 0.98, \quad P(0, 2) = 0.95, \quad P(0, 3) = 0.91, \quad P(0, 4) = 0.85.$$ 

The volatility of the two-year forward price on a zero-coupon bond with one year left to maturity is $0.05$. What is the Black price of a year $-3$ caplet with with cap rate equal to $0.04$ and $\$100$ notional amount?