Problem 6.1. (2 pts) Let the stock price $S(t)$ be modeled using the lognormal distribution. Define $Y(t) = S(t)^3$. Then, the random variable $Y(t)$ is lognormal itself. True or false?

Solution: TRUE

Problem 6.2. (2 pts) Let the stochastic process $S = \{S(t), t \geq 0\}$ represent the stock price as in the Black-Scholes model. Let its volatility term be denoted by $\sigma$. Then, the volatility parameter of the process $Y(t) = 2S(t)$ is $4\sigma$.

Solution: FALSE

Note: The correct answer is relevant to the solution for the Sample MFE Problem #54. The volatility parameter of the process $Y$ is $\sigma$.

Problem 6.3. (5 points) Solve Sample MFE Problem #33.

Solution: (c)

Problem 6.4. (2 pts) Theta may also called time decay. True or false?

Solution: TRUE

Problem 6.5. (5 points) There are two stocks present in our market: $S$ and $Q$. Their current prices are $S(0) = 60$ and $Q(0) = 65$. Both stocks pay dividends continuously. The dividend yield for $S$ is 0.02 while the dividend yield for $Q$ equals 0.03.

You are given that for $t \geq 0$

$$\text{Var}[\ln(S(t)/Q(t))] = 0.04t.$$  

What is the Black-Scholes price of a one-year exchange call with underlying $S$ and the strike asset $Q$?

(a) $2.86 
(b) $3.01 
(c) $7.27 
(d) $7.86 
(e) None of the above.

Solution: (b)

In our usual notation, the volatility of the difference of the stocks' realized returns is $\sigma = 0.2$. So,

$$d_1 = \frac{1}{0.2} \left[ \ln \left( \frac{60}{65} \right) + \left( 0.03 - 0.02 + \frac{0.04}{2} \right) \right]$$

$$= 5 \left[ \ln \left( \frac{60}{65} \right) + 0.03 \right] = -0.25,$$

$$d_2 = -0.25 - 0.2 = -0.45.$$

So,

$$N(d_1) = 1 - N(0.25) = 0.4013, \quad N(d_2) = 1 - N(0.45) = 0.3264.$$
Finally,

\[ V_{EC}(0, S, Q) = 60e^{-0.02(0.4013)} - 65e^{-0.03(0.3264)} = 3.01225. \]

**Problem 6.6.** (5 pts) Which of the following gives the correct values for the delta and gamma of a single share of non-dividend-paying stock?

(a) \( \Delta = 1, \Gamma = 1 \)
(b) \( \Delta = 1, \Gamma = 0 \)
(c) \( \Delta = 0, \Gamma = 1 \)
(d) \( \Delta = 0, \Gamma = 0 \)
(e) None of the above.

**Solution:** (b)

**Problem 6.7.** (5 points) Assume the Black-Scholes framework as model for the price of a non-dividend-paying stock. What is the difference between the delta of a European call option and the delta of the otherwise identical put option?

(a) 0
(b) 1
(c) \( S(0) \)
(d) Not enough information is given to answer this question.
(e) None of the above.

**Solution:** (b)

Put-call parity.

**Problem 6.8.** (2 points) Rho measures the sensitivity of a portfolio to the changes in the applicable risk-free interest rate. True or false?

**Solution:** TRUE

**Problem 6.9.** (2 points) Market makers can insure themselves against possible losses by trading in the out-of-the-money options.

**Solution:** TRUE

**Problem 6.10.** (2 points) In order to both delta hedge and gamma hedge a position in a certain option, the market-maker must trade in another type of option (i.e., not only in the money-market and the underlying risky asset).

**Solution:** TRUE

**Problem 6.11.** (5 points) The price of a zero-coupon bond redeemable at time— for $1 is denoted by \( P(0, T) \). You are given the following bond prices

\[ P(0, 1) = 0.93, \quad P(0, 2) = 0.86, \quad P(0, 3) = 0.77, \quad P(0, 4) = 0.70. \]

The volatilities of the forward prices for delivery at time—k of the above bonds at one year to maturity equal are denoted by \( \sigma_k \). We are given

\[ \sigma_1 = 0.10, \quad \sigma_2 = 0.11, \quad \sigma_3 = 0.12. \]

What is the Black price of a $0.92-strike, two-year European call option on the bond with one year to maturity on the exercise date?

(a) $0.0968
(b) $0.058
(c) $0.0368

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(d) $0.028
(e) None of the above.

Solution: (c)

\[ V_C(0) = P(0, 2)[FN(d_1) - N(d_2)] \]

with

\[ F = \frac{P(0, 3)}{P(0, 2)} = \frac{77}{86} = 0.8953, \]

\[ d_1 = \frac{1}{0.11\sqrt{2}} \left[ \ln \left( \frac{0.8953}{0.92} \right) + \frac{2 \times 0.11^2}{2} \right] = -0.0968, \]

\[ d_2 = d_1 - \sqrt{2}\sigma = -0.0968 - 0.11\sqrt{2} = -0.2524. \]

Using the standard normal tables, we obtain

\[ N(d_1) = 1 - N(0.10) = 0.4602, \quad N(d_2) = 1 - N(0.25) = 1 - 0.5987 = 0.4013. \]

Finally,

\[ V_C(0) = 0.77 \times 0.4602 - 0.92 \times 0.86 \times 0.4013 = 0.0368. \]

Provide a complete solution to the following problem(s):


Consider two assets with prices recorded as processes \( S \) and \( Q \) and an exchange call with \( S \) as the price of the underlying asset and \( Q \) as the price of the strike asset. The call’s maturity is \( T = 1 \) and the risk-free interest rate is \( r = 0.08 \).

Let \( S(0) = 40, \sigma_S = 0.3 \) and \( \delta_S = 0 \) and let \( Q(0) = 60, \sigma_Q = 0.5 \) and \( \delta_Q = 0 \). The correlation between the two prices is \( \rho = 1 \).

i. (6 pts) Find the price of the above described call.

ii. (6 pts) Change the volatility of the asset \( Q \) so that \( \sigma_Q = 0.4 \) Find the new price of the above described call.

iii. (4 pts) Find the connection between \( S \) and \( Q \) in both of the above cases, i.e., find a formula connecting the two prices.

iv. (2 pts) Using your answer to part iii., explain the effect on the price that you noticed in comparing the answers to part i. and part ii.

Solution:

i. Note that

\[ \sigma^2 = Var[\ln (S(t)/Q(t))] = 0.3^2 + 0.5^2 - 2 \times 0.3 \times 0.5 = 0.09 + 0.25 - 0.3 = 0.04. \]

So, we get \( \sigma = 0.2 \) to use in the Black-Scholes pricing formula. We get

\[ d_1 = \frac{1}{0.2} \left[ \ln \left( \frac{40}{60} \right) + \frac{0.04}{2} \right] = -1.93, \]

\[ d_2 = -1.93 - 0.2 = -2.13. \]

So, the price of the exchange call is

\[ V_{EC}(0) = S(0)N(d_1) - Q(0)N(d_2) = 40N(-1.93) - 60N(-2.13) = 0.07699. \]

ii. In this case, \( Var[\ln (S/Q)] = 0.3^2 + 0.4^2 - 2 \times 0.3 \times 0.4 = 0.01 \). Hence, we use a volatility of 0.1 in the Black-Scholes formula. With \( T = 1 \), we have the exchange option’s price equal to 0.000027405 – virtually zero.
iii. If $\ln(S)$ and $\ln(Q)$ are jointly normal with $\rho = 1$ (as in part (i.)), then they are linearly related. Hence

$$\ln(Q) = \ln(40) \left(1 - \frac{\sigma_Q}{\sigma_S}\right) + \frac{\sigma_Q}{\sigma_S} \ln(S). \quad (6.1)$$

In part (i.), $\ln(Q) = \ln(S) \implies Q = S$. For part (ii.),

$$\ln(Q) = -\frac{\ln(40)}{3} + \frac{4}{3} \ln(S) \implies Q = 0.2924S^{4/3}. \quad (6.2)$$

iv. If $S$ rises (say $S(T) = 50$) then $Q$ will be greater than $S$ (say $Q(T) = 53.861$); the option will be in the money if $S$ falls for $Q$ will fall by a greater amount making the exchange option have (real) value.