Loan repayment Schemes

L \cdot L \cdot i_{[0,T]} = \text{principal amount of interest}

In interest theory:
- amortized loan
- sinking fund
- catch all

Focus on interest-only loans

\Rightarrow \text{end of every period, the interest due is paid:}

\text{OLB}_{k+1} = \frac{R_{k-1}(k-1,k)}{L} \Rightarrow \text{for every k}

\Rightarrow \text{OLB}_{k+1} = \text{loan amt} = L

\Rightarrow \text{OLB}_{k+1} = L \cdot R_{k-1}(k-1,k)
Recall forward contracts

- underlying asset w/ price \( S(t), t \geq 0 \)
- fixed forward price \( F \)
- delivery date \( T \)

\[
\begin{array}{c}
\text{0} \\
\text{T}
\end{array}
\]

Unit of asset ↑ forward price

\[ \downarrow \]

Payoff (long forward) is

\[ S(T) - F \]

unheded hedge hedged

\[ -S(T) + S(T) - F = -F \]

In the case of stocks:

The forward price equals

\[
F_{0,T}(S) = \begin{cases} 
S(0) e^{rT} & \text{no dividends} \\
S(0) e^{(r-g)T} & \text{cont. dividends} \\
e^{rT}(S(0) - \sum \text{PV(dividends)}) & \text{discrete dividends}
\end{cases}
\]

Prices that are MODEL-FREE (only no ARBITRAGE!).
Swaps on interest rates:

\[ R \ldots \text{swap rate} \]
\[ \text{pmts @ end of period} \]
\[ L \cdot (R_{k-1}(k-1,k) - R) \]

"Good" hedge:

unhedged \hspace{1cm} \text{hedge} \hspace{1cm} \text{hedged}
\[ -L \cdot R_{k-1}(k-1,k) + L(R_{k-1}(k-1,k) - R) = -L \cdot R \]

w/ swap rate depending only on the term structure of interest rates.

Caps on interest rates:

\[ K_R \ldots \text{"strike" interest rate / cap rate} \]
\[ R_{k-1}(k-1,k) \]
\[ k-1 \quad k \]

Cap pmt @ time-\( k \):
\[ L \left( R_{k-1}(k-1,k) - K_R \right)_+ \]

Reality check:

unhedged \hspace{1cm} \text{hedge} \hspace{1cm} \text{hedged}
\[ -L \cdot R_{k-1}(k-1,k) + L(R_{k-1}(k-1,k) - K_R)_+ = -L \cdot \min(R_{k-1}(k-1,k), K_R) \]
Note: There MUST be a PREMIUM paid for the cap up front!

=> Must set up a model for Binomial Interest Rate Trees!

Odds-and-ends:
- If we "insure"/hedge for just one period, that financial instrument is called a **CAPLET**.

![Diagram with notation:](image)

- **Cap pmnt is made in arrears:**
  \[ L(R_{k-1}(k-1,k) - K_r)_+ \]

- If the cap pmnt is made in advance, then it equals:
  \[ L(R_{k-1}(k-1,k) - K_r)_+ \]
  \[ \frac{1 + R_{k-1}(k-1,k)}{1 + R_{k-1}(k-1,k)} \]