Logistics.

- HW #1:
  - prereqs
  - binomial c.r. trees

Due Wednesday, Jan 31st
(Signature sheets also!)

- Prereq In-Term: Jan 29th

Term structure of interest rates.

Observable:

- SPOT RATES: \( r_0(0, t) \) effective annual rates

\[ P(0, t) = \frac{1}{(1+r_0(0, t))^t} = (1+r_0(0, t))^{-t} \]

or

\[ r_0(0, t) = P(0, t)^{-\frac{1}{t}} - 1 \]
Forwards on Bonds.

- underlyng asset: zero-coupon; $1

- delivery date of the forward contract

- bond's maturity

F... the forward price for delivery @ time $T_F$
of a zero-coupon bond w/ $s = T - T_F$ left to maturity

REMEMBER: $F$ is paid on the delivery date $T_F$!

Consider: a prepaid forward analogous to the above forward; the only difference is that its price $F^P$ would be paid @ $t=0$.

$F^P = P(0, T)$

$F = FV_{0, T_F} (P(0, T))$

$= P(0, T) \left( 1 + r_0(0, T_F) \right)^{T_F}$

we use the given spot rates.
\[
F = \frac{P(0,T)}{P(0,T_F)}
\]

**Binomial interest-rate trees.**

**Interest-rate trees.**

*usually effective; yearly.*

**1-period**

\[ P_0 = P(0,1) = \frac{1}{1+r_0} \]

**2-periods**

\[ P_0 := P(0,2) = p \cdot \frac{1}{1+r_0} \cdot \frac{1}{1+r_u} + (1-p) \frac{1}{1+r_0} \cdot \frac{1}{1+r_d} \]

\( p \) ... RISK-NEUTRAL PROBABILITY, needs to be exponentially bounded
Recall: the stock-price trees

\[ S(0) \xrightarrow{\text{p}^*} S_u \quad \xleftarrow{1-\text{p}^*} S_d \]

\text{p}^* \text{ was defined to satisfy:}

\[ S(0)e^{rh} = p^* e^{rh} \cdot S_u + (1-p^*) e^{rh} \cdot S_d \]

\[ \Rightarrow \quad p^* = \frac{e^{(r-s)h} - d}{u-d} = \frac{S(0)e^{(r-s)h} - S_d}{S_u - S_d} \]
\[ P_0 = P(0,2) = \frac{1}{1+r_0} \left[ P \cdot \frac{1}{1+r_u} + (1-P) \cdot \frac{1}{1+r_d} \right] \]

\[ P(0,2) = P(0,1) \left[ P \cdot P_u + (1-P) \cdot P_d \right] \]

\[ \Rightarrow \quad p = \frac{\frac{P(0,2)}{P(0,1)} - P_d}{P_u - P_d} = \frac{E_{0,1} (P(1,2)) - P_d}{P_u - P_d} \]

Compare w/ the expression for \( p^* \) in the stock-price tree.