Problem 1.4. MFE Exam, Spring 2009: Problem #5.
You are given the following three-period interest rate tree. Each period is one year. The risk-neutral probability of each up-move is \( p = 70\% \). The interest rates are continuously compounded rates on the annual basis.

\[
\begin{align*}
P_{uu} &= e^{-r_{uu}} = 0.83527 \\
V_{uu} &= 0.06423 \\
V_{ud} &= 0.01308 \\
P_{ud} &= e^{-r_{ud}} = 0.88692 \\
P_{dd} &= e^{-r_{dd}} = 0.941765
\end{align*}
\]

Consider a European put option that expires in 2 years, giving you the right to sell a one-year zero-coupon bond for $0.90. This zero-coupon bond pays $1 at maturity. Determine the price of the put option.

\[
K = 0.90
\]

The put price:

\[
V_p(0) = e^{-r_0} \left[ p \cdot e^{-r_{uu}} \left( p \cdot V_{uu} + (1-p) \cdot V_{ud} \right) + (1-p) \cdot e^{-r_{dd}} \left( p \cdot V_{ud} + (1-p) \cdot V_{dd} \right) \right]
\]

\[
= 0.02853
\]
Caps.

Focus on the down-up path:

\[ V_{du} \times \frac{1}{1+r_{du}} \times \frac{1}{1+r_{d}} \times \frac{1}{1+r_{o}} \quad \text{w/ } p(1-p) \]

Focus on the up path:

\[ V_{u} \times \frac{1}{1+r_{u}} \times \frac{1}{1+r_{o}} \quad \text{w/ } \text{probabil. } p^2 \]
Problem 1.5. MFE Exam, Spring 2007: Problem #9.
You use a binomial interest rate model to evaluate a 7.5%-interest-rate cap on a $100 three-year loan. You are given

(i) The interest rates in the binomial tree are:
   \[ r_0 = 6\%, \quad r_u = 7.704\%, \quad r_{uu} = 9.892\%, \]
   \[ r_d = 4.673\%, \quad r_{ud} = r_{du} = 6\%, \]
   \[ r_{dd} = 3.639\%. \]

(ii) All interest rates are annual effective rates.
(iii) The risk-neutral probability that the annual effective interest rate moves up or down is 1/2.
(iv) The loan-interest payments are made annually.

Using the binomial interest rate model, calculate the time-0 value, i.e., the price, of this interest rate cap.

The cap's price:
\[
\frac{1}{1.06} \left[ \frac{1}{4} \times \frac{1}{1.09892} \times \frac{1}{1.07704} \times 0.02392 + \right.
\]
\[
+ \frac{1}{2} \times \frac{1}{1.07704} \times 0.00204 \left] \times 100 = \right.
\]
\[
= 0.57
\]

\text{(Buy the cap for 0.57.)}
With cap/caplet pmts:

\[
(L(1+r_m) - L(1+K_R))^+ =
\]

\[
= L(r_m - K_R)^+
\]

The analogous "insurance policy", i.e., a hedging instrument for the LENDER is a **floor** (or floorlet)

every pmt is

\[
L_x(K_R - R_{k-1, k})^+
\]

floor rate
• Model **EFFECTIVE** interest rates; usually annual.
• The risk-neutral probability is \( \frac{1}{2} \).

Every period after the first (trivial!) one has **two** parameters:

• \( R_k \) ... rate level parameter, i.e., the lowest i.r. @ that period \( k \)

• \( \sigma_k \) ... the volatility parameter of effective (annual) i.r.s.