Option Greeks

Descriptions of the sensitivity of the portfolios price/value as a function of one of these basic inputs/arguments:

- $s$ ... stock price (1st degree / 2nd degree) linear
- $t$ ... time
- $r$ ... continuously compounded interest rate
- $d$ ... dividend yield
- $\sigma$ ... volatility
- $K$ ... strike

Q: What happens to the portfolio's price when the value of one of the arguments changes w/ the values of all other arguments fixed?

**Delta**

... the first order sensitivity to perturbations in the stock price.

**Review:**

The call price in the binomial model.

**no dividends**

What's the equation for this line?
We need the intercept \& the slope.

The slope: \[ \Delta := \frac{V_u - V_d}{S_u - S_d} \] \# of shares to invest in to create a replicating portfolio.

The intercept: the risk-free investment \[ B e^{rT} \] in the replicating portfolio.

\[ \Rightarrow \] the equation for the line is:

\[ y = \Delta \cdot x + B e^{rT} \] \[ \Rightarrow \] Today's worth of the option is:

\[ V_c(s) = \Delta \cdot s + B \]

today's stock price

Notice: We can differentiate \( \Delta \) with respect to \( s \)!

We get:

\[ V_c'(s) = \Delta \]

This approach works better in the Black-Scholes setting!!!
Example. What is the $\Delta$ of a stock investment itself?

- The price of this portfolio, as a function of the CURRENT STOCK PRICE $s$, is:

$$\nu(s) = s$$
$$\Rightarrow \Delta(s) = \nu'(s) = \frac{\partial}{\partial s} \nu(s) = 1$$

Example. What is the $\Delta$ of a prepaid forward on the continuous dividend paying stock?

- The price of this derivative security, as a function of the CURRENT STOCK PRICE $s$:

$$\nu(s) = e^{-ST} \cdot s$$
$$\Rightarrow \Delta(s) = \nu'(s) = e^{-ST}$$

Example. The Black-Scholes call-option price:

$$\nu_c(s) = se^{-(T-t)} \left[ N(d_1) - Ke^{-(T-t)} N(d_2) \right]$$

\[ d_1 = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{s}{K} \right) + (r - \delta + \frac{\sigma^2}{2})(T-t) \right], \]

\[ d_2 = d_1 - \sigma \sqrt{T-t} \]

$$\Rightarrow \Delta_c(s) = ?$$

Product rule, chain rule, ....
\[ \Delta_c(s) = e^{-s(T-t)}N(d_1) \]

\[ \Rightarrow \text{For the put we get (using put-call parity):} \]
\[ \Delta_p(s) = -e^{-s(T-t)}N(-d_1) \]
8. You are considering the purchase of a 3-month 41.5-strike American call option on a nondividend-paying stock.

You are given:
(i) The Black-Scholes framework holds.
(ii) The stock is currently selling for 40.
(iii) The stock’s volatility is 30%.
(iv) The current call option delta is 0.5.

Determine the current price of the option.

\[ V_c(0) = S(0)N(d_1) - KE^{-rT}N(d_2) \]

\[ \Delta_c(S) = 0.5 \]

\[ N(d_1) = \Delta_c(S(0)) = 0.5 \]

(A) \( 20 - 20.453 \int_{-\infty}^{0.15} e^{-x^2/2} \, dx \)

(B) \( 20 - 16.138 \int_{-\infty}^{0.15} e^{-x^2/2} \, dx \)

(C) \( 20 - 40.453 \int_{-\infty}^{0.15} e^{-x^2/2} \, dx \)

(D) \( 16.138 \int_{-\infty}^{0.15} e^{-x^2/2} \, dx - 20.453 \)

(E) \( 40.453 \int_{-\infty}^{0.15} e^{-x^2/2} \, dx - 20.453 \)

We get \( r \) from

\[ N(d_1) = 0.5 \]

\( \Rightarrow \) \( d_1 = 0 \)

\( \Rightarrow \) \( \ln \left( \frac{40}{41.5} \right) + (r + 0.045) \cdot \frac{1}{4} = 0 \)

\( \Rightarrow \) \( r = 4 \cdot \ln \left( \frac{41.5}{40} \right) - 0.045 \)

\( r \approx 0.1023 \)

April 8, 2011
\[ d_2 = 0 - 0.3 \sqrt{\frac{1}{4}} = -0.15 \]
\[ N(d_2) = N(-0.15) = \int_{-\infty}^{0.15} \varphi(z) \, dz \]
\[ N(d_2) = 1 - N(0.15) = 1 - \int_{-\infty}^{0.15} \varphi(z) \, dz \]

\[ V_c(0) = 40 \cdot 0.5 - 41.5 e^{-0.1023 (0.25)} (1 - N(d_2)) \]
\[ 40.452 \]

\[ V_c(0) = 40.452 \left( \frac{N(d_2)}{0.15} \right) - 20.452 \]
\[ \int_{-\infty}^{0.15} \varphi(z) \, dz \]
\[ \varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \]

\[ \frac{40.452}{\sqrt{2\pi}} \cdot \int_{-\infty}^{0.15} e^{-\frac{z^2}{2}} \, dz \quad \Rightarrow \quad D. \]