
Eight months ago, an investor borrowed money at the risk-free interest rate to purchase a one-year 75-strike European call option on a nondividend-paying stock. At that time, the price of the call option was $8.

Today, the stock price is $85. The investor decides to close out all positions.

You are given:

(i) The continuously compounded risk-free rate interest rate is 5%.

(ii) The stock’s volatility is 26%.

Calculate the eight-month holding profit.

(A) 4.06

(B) 4.20

(C) 4.27

(D) 4.33

(E) 4.47
12. Answer: E

By (9.13), call price is a decreasing function of $K$. Thus, $C(50, T) \geq C(55, T)$. By the footnote on page 300,
\[ C(50, T) - C(55, T) \leq (55 - 50)e^{-rT}. \]
Thus, (I) is correct.

For (II) and (III), we start with their middle expression:
\[ P(45, T) - C(50, T) + S. \]
While there is not a direct relation between $P(45, T)$ and $C(50, T)$, we can use put-call parity to express $P(45, T)$ in terms of $C(45, T)$,
\[ P(45, T) - C(50, T) + S = [C(45, T) - S + 45e^{-rT}] - C(50, T) + S = C(45, T) - C(50, T) + 45e^{-rT}. \]
Similar to (I), we have
\[ 0 \leq C(45, T) - C(50, T) \leq (50 - 45)e^{-rT}, \]
which is equivalent to
\[ 45e^{-rT} \leq C(45, T) - C(50, T) + 45e^{-rT} \leq 50e^{-rT}. \]
Thus, (III) is correct.

Since (III) is correct, (II) must be incorrect.

13. Answer: A

8 months after purchasing the option, the remaining time to expiration = 4 months.
\[ d_1 = \frac{\ln(85/75) + (0.05 - 0 + \frac{1}{2} \times 0.26^2) 	imes 4/12}{0.26\sqrt{4/12}} = 1.019888 \approx 1.02, \ N(d_1) \approx 0.8461, \]
\[ d_2 = d_1 - \sigma\sqrt{T} = 1.019888 - 0.26\sqrt{4/12} = 0.869777 \approx 0.87, \ N(d_2) \approx 0.8078 \]
At time of purchase,
\[ C = SN(d_1) - Ke^{-rT}N(d_2) \approx 85 \times 0.8461 - 75e^{-0.05 \times (4/12)} \times 0.8078 = 12.3349 \]
Hence, 8-month holding profit is $12.3349 - 8e^{0.05 \times 8/12} = 4.0637 \approx 4.06$. 

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31. You compute the current delta for a 50-60 bull spread with the following information:

(i) The continuously compounded risk-free rate is 5%. 
(ii) The underlying stock pays no dividends. 
(iii) The current stock price is $50 per share. 
(iv) The stock’s volatility is 20%. 
(v) The time to expiration is 3 months. 

How much does delta change after 1 month, if the stock price does not change?

(A) increases by 0.04 
(B) increases by 0.02 
(C) does not change, within rounding to 0.01 
(D) decreases by 0.02 
(E) decreases by 0.04 

In general: 
\[ \Delta_{\text{call}} = e^{-STN(d_1)} \]
At $t=0$:
\[
\frac{d_1(50)}{\text{strike}} = \frac{1}{\sigma \sqrt{T}} \left[ \ln \left( \frac{50}{50} \right) + (0.05 + \left( \frac{0.2^2}{2} \right)) \cdot T \right]
\]
\[
= \frac{1}{0.2 \cdot \frac{1}{2}} \left( 0.05 + \frac{0.04}{2} \right) \cdot \frac{1}{4}
\]
\[
= 10 \left( 0.07 \right) \left( 0.25 \right) = 0.175
\]

\[N(d_1(50)) = N(d_1) = 0.5695 = \Delta \text{ of call w/ strike 50 (in the beginning)}\]

\[d_4(60) = \frac{1}{0.2 \cdot \frac{1}{2}} \left[ \ln \left( \frac{50}{60} \right) + (0.05 + 0.02) \cdot \frac{1}{4} \right] \]
\[
= 10 \left[ \ln \left( \frac{50}{60} \right) + \frac{0.07}{4} \right] \approx -1.648 \approx -1.645
\]

\[\Rightarrow N(d_4(60)) = 0.05\]

\[
\Rightarrow \Delta_{\text{Bull}} \text{ (at time 0 w/ } s(0) = 50) = 0.5695 - 0.05 = 0.5195
\]

\[\text{At } t = \frac{1}{12} \text{.}\]

We have $T-t = \frac{1}{4} - \frac{1}{12} = \frac{1}{6}$ to expiry.

\[
d_1(50) = \frac{1}{0.2 \sqrt{\frac{1}{6}}} \left[ \ln \left( \frac{50}{50} \right) + (0.07) \cdot \frac{1}{6} \right] = \frac{0.07}{0.2} \sqrt{\frac{1}{6}} \approx 0.14
\]

\[\Rightarrow N(d_1(50)) = 0.5557\]

\[
d_1(60) = \frac{1}{0.2 \sqrt{\frac{1}{6}}} \left[ \ln \left( \frac{50}{60} \right) + 0.07 \cdot \frac{1}{6} \right] = -2.09
\]

\[\Rightarrow N(d_4(60)) = 1 - N(2.09) = 0.0183\]

\[\Rightarrow \Delta_{\text{Bull}} \text{ (at time } t = \frac{1}{12} \text{ w/ } s(\frac{1}{12}) = 50) = 0.5557 - 0.0183 \approx 0.5374\]

\[\Delta \text{ went up by about 0.02 (a bit less)} \Rightarrow \]
The Gamma

\[ \Gamma = \text{second-order sensitivity w/ respect to the changes in the price of the underlying, i.e.,} \]
\[ \Gamma = \frac{\partial^2}{\partial s^2} \psi(s) \]

Q: What is the \( \Gamma \) of a stock investment?
\[ \psi''(s) = \frac{\partial^2}{\partial s^2} \psi(s) = 0 \]

Q: In the Black-Scholes model, what is the \( \Gamma \) of a call option?
\[ \Gamma_C(s) = \frac{\partial^2}{\partial s^2} \psi_C(s) = \frac{\partial}{\partial s} \left( \frac{\partial}{\partial s} \psi_C(s) \right) \]
\[ \Delta_C(s) \]
\[ \Gamma_C(s) = \frac{\partial}{\partial s} \left( e^{-r(T-t)} N(d_1) \right) \]
\[ \text{depends on } s \]
\[ \Gamma_C(s) = e^{-r(T-t)} \left( \frac{\partial}{\partial s} N(d_1) \right) \]
\[ \text{II \rightarrow chain rule} \]
\[ N'(d_1) \cdot \frac{\partial}{\partial s} d_1 \]
\[ \psi(d_1) \]
\( d_1 \) written explicitly as a function of \( s \):

\[
d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln(s) - \ln(K) + \left( r - \delta + \frac{\sigma^2}{2} \right) T \right]
\]

\[
\frac{\partial}{\partial s} d_1 = \frac{1}{\sigma \sqrt{T}} \cdot \frac{1}{3} = \frac{1}{3 \sigma \sqrt{T}}
\]

\[
\implies \Gamma_c(s) = e^{-8T} \cdot \phi(d_1) \cdot \frac{1}{3 \sigma \sqrt{T}}
\]