SAMPLE MFE

31. You compute the current delta for a 50-60 bull spread with the following information:

(i) The continuously compounded risk-free rate is 5%.

(ii) The underlying stock pays no dividends.

(iii) The current stock price is $50 per share.

(iv) The stock’s volatility is 20%. \( \sigma = 0.2 \)

(v) The time to expiration is 3 months. \( T = \frac{1}{4} \)

How much does delta change after 1 month, if the stock price does not change?

\[ t = \frac{1}{12} \Rightarrow T - t = \frac{1}{6} \]

\[ \Delta_{\text{Bull}} = \Delta_{50} - \Delta_{60} \]

(A) increases by 0.04

(B) increases by 0.02

(C) does not change, within rounding to 0.01

(D) decreases by 0.02

(E) decreases by 0.04

\[ d_1(50) = \frac{1}{\sigma \sqrt{T}} \left[ \ln\left(\frac{50}{50}\right) + (0.05 + \frac{\sigma^2}{2}) T \right] \]

\[ d_1(50) = \frac{1}{0.2 \cdot \frac{1}{2}} \left( 0.05 + \frac{0.04}{2} \right) \cdot \frac{1}{4} = \ldots = 0.175 \]

\[ N(d_1(50)) = 0.5695 \]

\[ d_1(60) = 1.0 \left[ \ln\left(\frac{50}{60}\right) + 0.07 \cdot \frac{1}{4} \right] = \ldots = -1.65 \]

\[ N(d_1(60)) = 0.05 \Rightarrow \]

\[ \Delta_{\text{Bull}}(0) = 0.5695 - 0.05 = 0.5195 \]
\[
\tilde{d}_1(50) = \frac{1}{\sigma \sqrt{T-t}} \left[ \ln \left( \frac{50}{60} \right) + 0.07 \cdot (T-t) \right] =
\]
\[
= \frac{0.07}{0.2} \sqrt{T-t} = \frac{7}{20} \cdot \sqrt{\frac{1}{6}} = \ldots = 0.14
\]
\[
\Rightarrow N(\tilde{d}_1(50)) \approx 0.56
\]
\[
\tilde{d}_4(60) = \frac{1}{0.2 \sqrt{\frac{1}{6}}} \left[ \ln \left( \frac{50}{60} \right) + 0.07 \cdot \frac{1}{6} \right] = -2.09
\]
\[
\Rightarrow N(\tilde{d}_4(60)) \approx 0.0183
\]
\[
\Delta_{\text{Bull}}(\chi_2) = 0.56 - 0.0183 \Rightarrow \text{about 0.02 increase} \Rightarrow \text{(B)}
\]
Option Elasticity (Black-Scholes)

\[ \Omega = \frac{S\Delta}{\mathcal{P}} \]

- For a call, we have

\[ S\Delta = Se^{-\delta T}N(d_1) > Se^{-\delta T}N(d_1) - Ke^{-r T}N(d_2) = C(S, \ldots) \]

- So, \( \Omega \geq 1 \)

- Similarly, for a put \( \Omega \leq \mathcal{P}^{-1} \)
41. Assume the Black-Scholes framework. Consider a 1-year European contingent claim on a stock.

You are given:

(i) The time-0 stock price is 45.
(ii) The stock’s volatility is 25%.
(iii) The stock pays dividends continuously at a rate proportional to its price. The dividend yield is 3%.
(iv) The continuously compounded risk-free interest rate is 7%.
(v) The time-1 payoff of the contingent claim is as follows:

\[ v(S) = \min(S, 42) = \text{bond} - \text{put} \]

![Payoff diagram](image)

Calculate the time-0 contingent-claim elasticity.

\[ \Omega = \frac{S(0) \cdot \Delta_0}{v(0)} \]

(A) 0.24  \[ v(0) = 42e^{-0.07} - V_p(0) \]

(B) 0.29  \[ v(0) = 42e^{-0.07} - (42e^{-0.07} \cdot N(-d_2) - 45e^{-0.03} \cdot N(d_2)) \]

(C) 0.34  \[ V(0) = 42e^{-0.07} \left( 1 - N(-d_2) \right) + 45e^{-0.03} \cdot N(-d_1) \]

(D) 0.39  \[ v(0) = 42e^{-0.07} \left( 1 - N(-d_2) \right) + 45e^{-0.03} \cdot N(-d_1) \]

(E) 0.44  \[ v(0) = 42e^{-0.07} \left( 1 - N(-d_2) \right) + 45e^{-0.03} \cdot N(-d_1) \]
\[ d_1 = \frac{1}{0.25\sqrt{T}} \left[ \ln\left(\frac{45}{42}\right) + (0.07 - 0.03 + \frac{(0.25)^2}{2}) \right] \]
\[ d_4 = 4 \left[ \ln\left(\frac{15}{11}\right) + (0.04 + \frac{(0.25)^2}{2}) \right] = \ldots = 0.56 \]
\[ d_2 = d_1 - \sigma\sqrt{T} = 0.56 - 0.25 = 0.31 \]
\[ V(0) = 42e^{-0.07} \cdot 0.6217 + 45e^{-0.03} \cdot (1 - 0.7123) \]
\[ V(0) = 36.80 \]
\[ \Delta = \Delta_{\text{bond}} - \Delta_{\text{put}} = + \left( e^{-8T} N(-d_1) \right) = e^{-0.03} N(-0.56) \]
\[ \Omega = \frac{45 \cdot e^{-0.03} (1 - 0.7123)}{36.8} \approx 0.34 \implies \text{C} \]
\[ V_c(s) - V_p(s) = s \Phi_0 e^{-sT} - PV(K) \] 

\[ \Delta_c - \Delta_p = e^{-sT} \]

\[ \Gamma_c - \Gamma_p = 0 \implies \Gamma_c = \Gamma_p \]
53. Assume the Black-Scholes framework. For a European put option and a European gap call option on a stock, you are given:

(i) The expiry date for both options is $T$.
(ii) The put option has a strike price of 40.
(iii) The gap call option has strike price 45 and payment trigger 40.
(iv) The time-0 gamma of the put option is 0.07. $\implies \Gamma_c = 0.07$
(v) The time-0 gamma of the gap call option is 0.08.

Consider a European cash-or-nothing call option that pays 1000 at time $T$ if the stock price at that time is higher than 40.

Find the time-0 gamma of the cash-or-nothing call option.

(A) $-5$
(B) $-2$
(C) 2
(D) 5
(E) 8

Idea: Replicate the cash call in terms of the given options.

\[
V_{cc}(T) = \mathbb{I}_{[S(T) > 40]}
\]

\[
V_c(T) = (S(T) - 40)_+ = (S(T) - 40)\mathbb{I}_{[S(T) > 40]}
\]

\[
V_{GC}(T) = (S(T) - 45)\mathbb{I}_{[S(T) > 40]}
\]

\[
V_c(T) - V_{GC}(T) = -40\mathbb{I}_{[S(T) > 40]} + 45\mathbb{I}_{[S(T) > 40]} = 5\mathbb{I}_{[S(T) > 40]} = 5 \cdot V_{cc}(T)
\]
\[
\left( \Gamma_c - \Gamma_{GC} = 5 \Gamma_{cc} \right) \div 200
\]

\[
200 \left( \Gamma_c - \Gamma_{GC} \right) = 1,000 \cdot \Gamma_{cc} = \text{answer}
\]

\[
200 (0.07 - 0.08) = -2 \Rightarrow \text{？}
\]