Assume that the binomial interest-rate tree is populated with the following effective annual interest rates:

\[ r_0 = 0.04, \quad r_u = 0.045, \quad r_d = 0.035. \]

We observe that a zero-coupon bond redeemable in two-years for $100 is priced at $P(0) = 92.5$ using the above interest-rate model. What is the risk-neutral probability $p$ stipulated by the model?
Problem 1.2. (i) The interest rates in the binomial tree are:
\[ r_0 = 6\%, \quad r_u = 7.704\%, \quad r_{uu} = 9.892\%, \]
\[ r_d = 4.673\%, \quad r_{ud} = r_{du} = 6\%, \]
\[ r_{dd} = 3.639\%. \]

(ii) All interest rates are annual effective rates.
(iii) The risk-neutral probability that the annual effective interest rate moves up or down is 1/2.
Find the price of a zero-coupon bond redeemable for $1,000 in three years.
Problem 1.3. Call on a bond.
Consider a two-period binomial interest-rate tree. Assume that the root effective annual interest rate equals \( r_0 = 0.05 \). From then on, the interest rates change according to an up factor equal to \( u = 1.2 \) and the down factor \( d = 0.08 \).

Find the price of a zero-coupon bond redeemable for $1 in two years.

Next, price a call option on the above bond with exercise date at time \( T_o = 1 \) and with strike \( K = 0.95 \).
Problem 1.4. MFE Exam, Spring 2009: Problem #5.
You are given the following three-period interest rate tree. Each period is one year. The risk-neutral probability of each up-move is \( p = 70\% \). The interest rates are **continuously compounded** rates on the annual basis.

Consider a European put option that expires in 2 years, giving you the right to sell a one-year zero-coupon bond for $0.90. This zero-coupon bond pays $1 at maturity. Determine the price of the put option.
Problem 1.5. MFE Exam, Spring 2007: Problem #9.

You use a binomial interest rate model to evaluate a 7.5%-interest-rate cap on a $100 three-year loan. You are given

(i) The interest rates in the binomial tree are:
\[ r_0 = 6\%, \quad r_u = 7.04\%, \quad r_{uu} = 9.892\%, \]
\[ r_d = 4.673\%, \quad r_{ud} = r_{du} = 6\%, \]
\[ r_{dd} = 3.639\%. \]

(ii) All interest rates are annual effective rates.

(iii) The risk-neutral probability that the annual effective interest rate moves up or down is 1/2.

(iv) The loan-interest payments are made annually.

Using the binomial interest rate model, calculate the time−0 value, i.e., the price, of this interest rate cap.