Notes: This is a closed book and closed notes exam.
Time: 50 minutes

### TRUE/FALSE

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### MULTIPLE CHOICE

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### FOR GRADER'S USE ONLY:

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2.1. **TRUE/FALSE QUESTIONS.** Please note your answers on the front page.

**Problem 2.1.** Continuously compounded returns of stocks are multiplicative.

**Problem 2.2.** Let $X$ be a strictly positive random variable, then
\[ \mathbb{E}[X] = |\mathbb{E}[X]|. \]

**Problem 2.3.** (2 points)
Let $X$ be a random variable whose variance is zero. Then $X$ is constant with probability one.

**Problem 2.4.** In the lognormal stock-price model, the mean time $-t$ stock price is increasing as a function of $t$.

2.2. **FREE-RESPONSE PROBLEMS.** Please, explain carefully all your statements and assumptions. Numerical results or single-word answers without an explanation (even if they’re correct) are worth 0 points.

**Problem 2.5.** (10 points) Let the stock prices be modeled using the lognormal distribution. The mean stock price at time $-1$ equals 120 and the median stock price 115. What is the probability that the time $-1$ stock price exceeds 100?

2.3. **MULTIPLE CHOICE QUESTIONS.**

**Problem 2.6.** A continuous-dividend-paying stock has a dividend yield of 0.02. The time $-t$ realized (rate of) return is modeled as
\[ R(0, t) \sim N(\text{mean} = 0.03t, \text{variance} = 0.09t) \]

Find the probability that the time $-4$ stock price exceeds today’s stock price.

(a) 0.4920  
(b) 0.4960  
(c) 0.5040  
(d) 0.5080  
(e) None of the above.
Problem 2.7. The current price of continuous-dividend-paying stock is $100 per share and its volatility is 0.25. The stock price is modeled using a log-normal distribution. The expected time−2 stock price is $120.

Then, the median of the time−2 stock price falls within this interval:

(a) [0, 86]
(b) [86, 106]
(c) [106, 112]
(d) [112, 124]
(e) None of the above.

Problem 2.8. (5 pts)
A non-dividend-paying stock is currently valued at $100 per share. Its annual mean rate of return is given to be 12% while its volatility is given to be 30%.

Assuming the lognormal stock-price model, find

\[ \mathbb{E}[S(2) \mid S(2) > 95]. \]

(a) $86.55
(b) $101.60
(c) $152.35
(d) $159.07
(e) None of the above.

Problem 2.9. (5 pts) Assume the Black-Scholes framework. Let the current price of a non-dividend-paying stock be equal to \( S(0) = 95 \) and let its volatility be equal to 0.35. Consider a European call on that stock with strike 100 and exercise date in 9 months. Let the risk-free continuously compounded interest rate be 6% per annum.

Denote the price of the call by \( V_C(0) \). Then,

(a) \( V_C(0) < 5.20 \)
(b) \( 5.20 \leq V_C(0) < 7.69 \)
(c) \( 7.69 \leq V_C(0) < 9.04 \)
(d) \( 9.04 \leq V_C(0) < 11.25 \)
(e) None of the above.

Problem 2.10. (5 pts) Assume the Black-Scholes framework. Let the current price of a share of stock be equal to \( S(0) = 80 \), let its volatility be \( \sigma = 0.3 \), and let \( \delta = 0 \) (in our usual notation).

Consider a gap option with expiration date \( T = 1 \) year such that its payoff is \( S(T) - 90 \) if \( S(T) > 100 \).

You are given that the continuously compounded risk-free interest rate equals \( r = 0.05 \) per annum.

Let \( V_{GC}(0) \) denote the price of the above gap option. Then,

(a) \( V_{GC}(0) < 3.20 \)
(b) \( 3.20 \leq V_{GC}(0) < 5.69 \)
(c) $5.69 \leq V_{GC}(0) < 7.04$
(d) $7.04 \leq V_{GC}(0) < 11.25$
(e) None of the above.

Problem 2.11. Assume the Black-Scholes setting.
Mary wagers to pay one share of stock to Matt if the price at expiration in 1 year is above $75.00.
Assume $S(0) = 60.00, \sigma = 0.15, r = 0.04$, and the dividend rate of 0.01. What is the value of Marys bet?
(a) 6.72
(b) 7.52
(c) 8.72
(d) 9.51
(e) None of the above.

Problem 2.12. Assume the Black-Scholes setting.
Assume $S(0) = 63.75, \sigma = 0.20, r = 0.055$. The stock pays no dividend and the option expires in 50 days (simplify the number of days in a year to 360).
What is the price of a $60$-strike European put?
(a) 0.66
(b) 0.55
(c) 0.44
(d) 0.37
(e) None of the above.

Problem 2.13. Assume the Black-Scholes setting.
Assume $S(0) = 28.50, \sigma = 0.32, r = 0.04$. The stock pays a 1.0% continuous dividend and the option expires in 110 days (simplify the number of days in a year to 360).
What is the price of a $30$-strike put?
(a) 2.75
(b) 2.10
(c) 1.80
(d) 1.20
(e) None of the above.

Problem 2.14. (5 points) Consider a non-dividend-paying stock whose price $S = \{S(t), t \geq 0\}$ is modeled using a geometric Brownian motion. Suppose that the current stock price equals $100 and that its volatility is given to be 0.25.
The continuously compounded, risk-free interest rate is assumed to equal 0.04.
Consider a derivative security which entitles its owner to obtain a European call option on the above stock six months from today, i.e., at time $t_* = 1/2$. The call option is to be 3–month to expiration at time of delivery and have the strike equal to 105% of the time–$t^*$ price of the underlying asset. This contract is called a forward start option.
What is the price of the forward start option?
(a) 3.15
(b) 8.65
(c) 10.51
(d) 13.55
(e) None of the above.