Option Greeks

1 Introduction
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Set-up

- **Assignment:** Read Section 12.3 from McDonald.
- We want to look at the option prices dynamically.
- **Question:** What happens with the option price if *one* of the inputs (parameters) changes?
- First, we give names to these effects of perturbations of parameters to the option price. Then, we can see what happens in the contexts of the pricing models we use.
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Vocabulary
• \( \Psi \) is rarer and denotes the sensitivity to the changes in the dividend yield \( \delta \)
• *vega* is not a Greek letter - sometimes \( \lambda \) or \( \kappa \) are used instead
• The “prescribed” perturbations in the definitions above are problematic . . .
• It is more sensible to look at the Greeks as *derivatives* of option prices (in a given model)!
• As usual, we will talk about calls - the puts are analogous
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The Delta: The binomial model

- Recall the replicating portfolio for a call option on a stock $S$: $\Delta$ shares of stock & $B$ invested in the riskless asset.
- So, the price of a call at any time $t$ was
  \[ C = \Delta S + Be^{rt} \]
  with $S$ denoting the price of the stock at time $t$
- Differentiating with respect to $S$, we get
  \[ \frac{\partial}{\partial S} C = \Delta \]
- And, I did tell you that the notation was intentional ...
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The Delta: The Black-Scholes formula

- The Black-Scholes call option price is

\[ C(S, K, r, T, \delta, \sigma) = Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2) \]

with

\[ d_1 = \frac{1}{\sigma \sqrt{T}} \left[ \ln\left( \frac{S}{K} \right) + (r - \delta + \frac{1}{2} \sigma^2) T \right], \quad d_2 = d_1 - \sigma \sqrt{T} \]

- Calculating the \( \Delta \) we get . . .

\[ \frac{\partial}{\partial S} C(S, \ldots) = e^{-\delta T} N(d_1) \]

- This allows us to reinterpret the expression for the Black-Scholes price in analogy with the replicating portfolio from the binomial model
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The Gamma

- Regardless of the model - due to put-call parity - $\Gamma$ is the same for European puts and calls (with the same parameters)

- In general, one gets $\Gamma$ as
  \[
  \frac{\partial^2}{\partial S^2} C(S, \ldots) = \ldots
  \]

- In the Black-Scholes setting
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  \frac{\partial^2}{\partial S^2} C(S, \ldots) = \frac{e^{-\delta T - 0.5d_1^2}}{S\sigma\sqrt{2\pi T}}
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- If $\Gamma$ of a derivative is positive when evaluated at all prices $S$, we say that this derivative is convex
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The Vega

- Heuristically, an increase in volatility of $S$ yields an increase in the price of a call or put option on $S$
- So, since \( \text{vega} \) is defined as
  \[
  \frac{\partial}{\partial \sigma} C(\ldots, \sigma)
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  we conclude that \( \text{vega} \geq 0 \)
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The Theta

- When talking about $\theta$ it is more convenient to write the parameters of the call option’s price as follows:

$$C(S, K, r, T - t, \delta, \sigma)$$

where $T - t$ denotes the time to expiration of the option.

- Then, $\theta$ can be written as

$$\frac{\partial}{\partial t} C(\ldots, T - t, \ldots)$$

- What is the expression for $\theta$ in the Black-Scholes setting?

- Caveat: It is possible for the price of an option to increase as time to expiration decreases.
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The Rho

- \( \rho \) is defined as

\[ \frac{\partial}{\partial r} C(\ldots, r, \ldots) \]

- In the Black-Scholes setting,

\[ \rho = KTe^{-rT} N(d_2) \]

- It is not accidental that \( \rho > 0 \) regardless of the values of the parameters: when a call is exercised, the strike price needs to be paid and as the interest rate increases, the present value of the strike decreases.

- In analogy, for a put, \( \rho < 0 \).
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The Psi

- $Ψ$ is defined as

$$\frac{∂}{∂δ} C(\ldots, δ, \ldots)$$

- What is the expression for $Ψ$ in the Black-Scholes setting?
- You should get that $Ψ < 0$ for a put - regardless of the parameters. The reasoning justifying this is analogous to the one for $ρ$: when a call is exercised, the holder obtains shares of stock - but is not entitled to the dividends paid prior to exercise and, thus, the present value of the stock is lower for higher dividend yields.
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The Psi

- $\Psi$ is defined as
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Option Elasticity (Black-Scholes)

- For a call, we have

$$S\Delta = Se^{-\delta T} N(d_1) > Se^{-\delta T} N(d_1) - Ke^{-rT} N(d_2) = C(S, \ldots)$$

- So, $\Omega \geq 1$
- Similarly, for a put $\Omega \leq 0$
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