HW Assignment 5

Problem 5.1. Suppose that ABC is a non-dividend-paying stock whose price is modeled using the Black-Scholes pricing assumptions. Assume that its spot price is $100, the volatility is 0.4. Let $r = 0.06$.

a. (5 pts) What is the price of a European call option with 105 strike and 1 year to expiration?

b. (5 pts) What is the 1-year forward price on the ABC stock?

c. (5 pts) What is the price of a European call option with strike 105 and maturity 1 year for which the underlying asset is a futures contact whose maturity is the same as that of the option, i.e., 1 year?

Problem 5.2. (10 points) Imagine that a fairly regular ball of salt has been immersed in water. We want to model the rate of decrease in the mass of the ball due to the salt getting diffused in water. Somebody proposes that at any time this rate should be proportional to the exposed surface of the ball – so that the salt melts away “layer by layer”. This is plausible enough. The exposed surface of the ball is proportional to the square of the radius. The mass of the ball is proportional to the cube of the radius. So, the proposed model would state that rate of change of the mass of the ball is proportional to the current mass of the ball at that time raised to the power $2/3$. Let the function $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ represent the mass of the ball as a function of time. Let the initial mass be $m_0 > 0$. Write down the ordinary differential equation for the function $f$ corresponding to the above proposed model.

Problem 5.3. Harry has a savings account earning interest at the continuously compounded interest rate $r$. The initial balance in this account is denoted by $b_0 > 0$.

Over the course of the following year, Harry continuously withdraws funds from this account at a rate which is at any time equal to half of the balance in the account at that time. Let the function $B : \mathbb{R}_+ \rightarrow \mathbb{R}$ represent the balance in Harry’s account as a function of time.

(i) (10 points) Write down the ordinary differential equation satisfied by the function $B$. Do not forget the initial condition!

(ii) (10 points) Solve the ordinary differential equation you obtained in part (i).

Problem 5.4. (15 points) Consider the function $F : \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}$ of two variables defined as $F(x, t) = e^{t/2} \sin(x)$. Find the following partial derivatives of this function:

(i) $F_t(x, t) = \frac{\partial F(x, t)}{\partial t}$

(ii) $F_x(x, t) = \frac{\partial F(x, t)}{\partial x}$

(ii) $F_{xx}(x, t) = \frac{\partial^2 F(x, t)}{\partial x^2}$

Provide your final answer only for the following problem(s).

Problem 5.5. (1 pt) For purposes of Black-Scholes option pricing, when the movement of a stock price follows a geometric Brownian motion, the stock price is said to follow which type of distribution?

(a) Weibull
(b) Normal
(c) Lognormal
(d) Binomial
(e) None of the above.