3.1. **The uniform distribution.** Let $a < b$ be two real numbers. We say that a random variable $X$ is **uniformly distributed** on the interval $[a, b]$ if it has the following probability density function

$$f_X(x) = \frac{1}{b-a} \mathbb{I}_{[a,b]}(x)$$

where $\mathbb{I}$ denotes the indicator function, i.e.,

$$\mathbb{I}_{[a,b]}(x) = \begin{cases} 
1 & \text{if } a \leq x \leq b \\
0 & \text{otherwise.} 
\end{cases}$$

For short, we write $X \sim U(a, b)$. In particular, a random variable which is uniformly distributed on the interval $[0, 1]$ is referred to as the **unit uniform** random variable. We frequently use $U$ to denote a unit uniform random variable. If there are multiple (oftentimes independent!) random variables with the same unit uniform distribution, we introduce indices and write

$$U_1, U_2, \ldots, U_n, \ldots \sim U(0, 1).$$

It is easy to show that

$$\mathbb{E}[U] = 1/2 \quad \text{and} \quad \text{Var}[U] = 1/12.$$  

The above principle can be generalized to construct uniform distributions in multiple dimensions and on more complex domains.

3.2. **Random number generation.** We intend to use unit uniform random variables as building blocks for more complicated distributions. More precisely, our next task is to attempt to use software to produce sets of values which seemingly come from a specific distribution. There are two steps to completing this task:

1. First, we need to convince ourselves that computers can produce streams of numbers that are uniformly distributed in $[0, 1]$.
2. Second, we show how the above streams of numbers should be modified to produce other streams of values that seemingly come from a different distribution. More precisely, we need to figure out which function needs to be applied to the draws from a uniform distribution so that the results appear to have a specific distribution.

Despite the fact that the first step is an incredibly interesting topic, it is beyond the scope of this course. We will simply accept the fact that any reasonable software will be able to produce draws from a uniform distribution. An ambitious reader will, however, spend some time thinking about why this question is interesting, what the obstacles are, and what criteria one might use to judge whether a set of outputs is “random enough” and/or “uniform enough”.

**Instructor:** Milica Ćudina  
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