Problem Set # 4

Bernoulli. Binomial.

Provide your complete solutions for the following problems.

Problem 4.1. Based on the traveling salesman’s experience, he makes a same on any visit with probability of 15%. We assume that the individual customer’s decisions are independent.

If he makes 10 visits in a certain day, what is the chance that he makes at least five sales?

Solution: Let $Y$ denote the number of sales he makes. Then $Y \sim \text{Binomial}(n = 10, p = 0.15)$.

Note $P(Y \geq 5] = 1 - P[Y \leq 4].$

So,$P[Y \leq 4] = P[Y = 0] + \cdots + P[Y = 4]$

$= \binom{10}{0}(0.15)^0(0.85)^{10} + \binom{10}{1}(0.15)^1(0.85)^9 + \binom{10}{2}(0.15)^2(0.85)^8 + \binom{10}{3}(0.15)^3(0.85)^7 + \binom{10}{4}(0.15)^4(0.85)^6$

$= \cdots = 0.99012.$

The answer is $P(Y \geq 5] = 0.0099.$

Problem 4.2. Expected frequency

Suppose you are going to roll a fair die 60 times and record the proportion of times that a 1 or a 2 is showing. The sampling distribution of the said proportion should be centered about which value?

Solution: $1/3$

We model every roll in the repeated $n$–tuple of trials as a single Bernoulli trial. So, in this particular problem, we have $X_i, i = 1, \ldots, n$ to be independent, identically distributed with $X_i \sim \text{Bernoulli}(p = 1/3).$

Note that $E[X_i] = p = 1/3$ for all $i$.

The statistics denoting the proportion of “successes” in the repeated trials is defined as

$\hat{p} = \frac{X_1 + X_2 + \cdots + X_n}{n}.$

The center of the sampling distribution of $\hat{p}$ is exactly its expected value. So, in the present problem,

$E[\hat{p}] = \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{nE[X_1]}{n} = E[X_1] = p = 1/3.$