Provide your **complete solution** for the following problem(s).

**Problem 5.1.** (5 points) Source: “Probability and Statistics for Engineers and Scientists” by Walpole, Myers, Myers, and Ye.

The probability that a patient recovers from a rare blood disease is 0.4. It is known that 100 people have contracted the said disease. Survival between patients is independent. What is the approximate probability that fewer than 30 survive?

*For practice:* Use the continuity correction!

**Solution:** Let $X$ denote the number of survivors in the cohort of 100. Then, $X \sim \text{Binomial}(n = 100, p = 0.4)$. To use the normal approximation to the binomial distribution, we need the parameters

$$
\mu = np = 40, \quad \sigma = \sqrt{100(0.4)(0.6)} \approx 4.899.
$$

With the continuity correction, we have

$$
P[X < 30] = P[X \leq 29.5] = P \left[ \frac{X - \mu}{\sigma} \leq \frac{29.5 - 40}{4.899} \right] \approx \Phi(-2.14) = 0.0162.
$$

**Problem 5.2.** (7 points) Source: “Probability and Statistics for Engineers and Scientists” by Walpole, Myers, Myers, and Ye.

Consider a multiple-choice exam consisting of 200 questions, each with four possible answers out of which only one is correct. Each problem is worth one point. The student comes into the exam exactly knowing 120 out of the 200 questions. The student intends to guess at random the answers to the remaining 80 questions.

What is the approximate probability that the student’s final score in the exam is between 145 and 150 inclusive?

*For practice:* Use the continuity correction!

**Solution:** Let $X$ denote the number of answers guessed correctly out of the 80 “unknown” questions. Then, $X \sim \text{Binomial}(n = 80, p = 0.25)$. To use the normal approximation to the binomial distribution, we need the parameters

$$
\mu = np = 20, \quad \sigma = \sqrt{np(1-p)} = \sqrt{80(0.25)(0.75)} = 3.873.
$$

The probability we seek is

$$
P[25 \leq X \leq 30] = P[24.5 \leq X \leq 30.5] = P \left[ \frac{24.5 - 20}{3.873} \leq \frac{X - \mu}{\sigma} \leq \frac{30.5 - 20}{3.873} \right] \approx \Phi(2.71) - \Phi(1.16) = 0.1196.
$$
Problem 5.3. (3 points)

Let $X$ be the count of successes in $n$ Bernoulli trials, each with probability of success equal to $p$. Assume, that $n \geq 30$, $np > 10$ and $n(1-p) > 10$.

Then, the random variable $\hat{P} = X/n$ stands for the proportion of successes in the $n$ trials. Using the normal approximation to the binomial distribution, what do you conclude the approximate distribution of $\hat{P}$ must be?

Solution:

We approximate the distribution of $X$ by

$$N(\text{mean} = np, \text{variance} = np(1-p)).$$

Therefore, the approximate distribution of $\hat{P}$ is

$$N(\text{mean} = p, \text{variance} = p(1-p)/n).$$