Provide your complete solution for the following problems.

**Problem 2.1.** (9 points) *Source: “Probability” by Jim Pitman.*

Consider the sample average $\bar{X}_n$ of $n$ independent random variables, each of them uniformly distributed on $[0, 1]$. Find the smallest sample size $n$ such that $P[\bar{X}_n < 0.51]$ is at least 90%. Use the Central Limit Theorem.

**Problem 2.2.** *Source: “Probability” by Pitman.*

Suppose that on average 1 in 100 people gave a certain genetic marker.

(i) (3 points) Suppose a simple random sample of 50 is selected and tested. What is the probability that at least one of them will have the genetic marker?

(ii) (3 points) About how many people would have to be tested so that the probability of having at least one positive result would be at least 99%? If this number of people were tested, what is the expected number of positives?