Put-call parity: The general case

8.1. Construction. Let Portfolio A consist of a long European call and a short European put on the same underlying asset $S$ with the same strike $K$ and the same exercise date $T$. The initial value of this portfolio is

$$V_A(0) = V_C(0) - V_P(0).$$

There are no intermediate cash-flows associated with this portfolio and its payoff at time $T$ is

$$V_C(T) - V_P(T) = S(T) - K.$$

On the other hand, let Portfolio B consist of the following:

1. a long prepaid forward contract on $S$ for delivery at time $T$,
2. borrowing the present value of the strike price to be repaid at time $T$.

Then, the initial cost of this portfolio equals:

$$F_{0,T}^P(S) - PV_{0,T}(K).$$

Since there are no intermediate cash-flows associated with this portfolio, either, its payoff at time $T$ is

$$S(T) - K.$$

Since the above portfolios have the same final payoff, by the no-arbitrage principle, we conclude that their initial values must also be the same. We get the more general version of put-call parity:

$$V_C(0) - V_P(0) = F_{0,T}^P(S) - PV_{0,T}(K).$$

8.2. Special cases. Our most common setting is the one with a continuously compounded interest rate $r$. In that case the put-call parity reads as

$$V_C(0) - V_P(0) = F_{0,T}^P(S) - Ke^{-rT}.$$

With respect to dividends, these are the three cases we will be looking into:

- non-dividend-paying stocks:
  $$V_C(0) - V_P(0) = S(0) - Ke^{-rT}$$

- discrete dividends $D_i, i = 1, \ldots, n$ at times $0 < t_1 < \cdots < t_n \leq T$:
  $$V_C(0) - V_P(0) = S(0) - \sum_{i=1}^{n} D_i e^{-rt_i} - Ke^{-rT}$$

- continuous dividends at the rate $\delta$:
  $$V_C(0) - V_P(0) = S(0)e^{-\delta T} - Ke^{-rT}$$
8.3. MFE Exam Spring 2007: Problem #1. On April 30, 2007, a common stock is priced at $52.00. You are given that:

(1) Dividends in equal amounts are to be paid on June 30, 2007, and on September 30, 2007.

(2) A European call on the above stock with strike $K = 50$ and the exercise date in six months sells for $4.50.

(3) A European put on the above stock with strike $K = 50$ and the exercise date in six months sells for $2.45.

(4) The continuously-compounded risk-free interest rate equals 0.06.

Calculate the amount of each dividend.