3.1. The time-line. Before we look into specific derivatives, let us introduce two interesting functions of the price of the underlying asset: the payoff and the profit. The first part of the course focuses almost exclusively on the study of the payoff and the profit structures of a small set of derivative securities.

Let us start with a simple time-line. We choose the beginning of time, i.e., time $-0$, so that it suits our purposes. Most often, this is the time at which the contract is “signed” – the agreement between the two parties trading the derivative is reached. We frequently refer to time $-0$ as “today”. Using the notation from the first lecture, the price of the underlying asset today is denoted by $S(0)$.

Every one of the derivatives we look into will stipulate a precise date $T$ by which all of the cash-flows (payments) associated with the derivative must take place. Depending on the type of the derivative, $T$ is referred to as the delivery date, expiration date, maturity, exercise date, . . . The terms are traditional and are frequently used interchangeably even though some of them make more sense in certain settings than in other. The random variable denoting the price of the underlying asset at time $T$ will be denoted by $S(T)$. We are not clairvoyant, so we cannot tell in advance what this price is going to be. But, we can look at how the cash-flows associated with a certain derivative behave as a specific function $v$ of $S(T)$ or in a more general situation, as a function of a subset of observed prices $\{S(t), t \in [0, T]\}$. More precisely, we can look at how the exact monetary amounts of those cash-flows stipulated in the contract depend on $S(T)$, and we will display the values $s$ on the positive part of the $x$–axis.

3.2. The two sides. The contracts we intend to look at will all have two sides:

- the buyer/holder of the derivative (the long position), and
- the seller/writer of the derivative (the short position).

Regardless of the actual amounts of cash-flows associated with a certain derivative, the following are true in most cases:

- the initial cash-flows at time $-0$ (if any) go from the buyer to the writer of the derivative,
- the final and intermediate cash-flows (if any) can go both from the writer to the buyer and vice versa.

It is clear that we can focus on just one side of the contract and consider that side’s cash-flows only. If this is the buyer’s side, then the initial cash-flow will be nonpositive. The (possibly) negative sign indicates that the buyer paid out the required amount (the price) to enter the contract. This amount might be zero, but it will not be negative (i.e., the buyer will not receive the money to enter the contract). We can denote this amount (the value of the derivative security) by $V(0)$. We also refer to this monetary amount as the initial cost of entering the contract. Ignoring for the moment the possible intermediate cash-flows, the final cash-flow, i.e., the payoff of the derivative for the buyer might be any real number (depending on $S(T)$). This payoff is one of the defining properties of the derivative security.
In fact, once the payoff function is applied to the random variable $S(T)$, the result is the value of the derivative security $V(T)$ we discussed previously.

To the contrary, if we focus on the writer’s side, the situation is symmetric in the sense that the cash amounts paid out by the buyer are received by the seller and vice versa. So, the writer initially obtains the monetary amount $V(0) \geq 0$. At time $-T$, the writer’s payoff is $-V(T)$.

3.3. The payoff curve. The above set-up allows us to study the payoffs of both the buyer and the seller as functions of the final asset price $S(T)$. We will usually draw graphs to describe this dependence better. Below is an example of a payoff curve.

![Payoff Curve Graph](image)

The $x$–axis corresponds to the payoff function’s argument $s$ (the “placeholder” for the possible values of $S(T)$).

3.4. The profit curve. Going back to the reasons for introducing derivative securities, we see that some of them are supposed to serve as a kind of insurance policies for the buyer. It is possible (and it is oftentimes the case) that the buyer’s payoff is never negative and it is strictly positive for certain values of the argument. This means that the writer of the derivative is required to pay out a certain amount of money to the buyer. They would want to be compensated for this – this is the reason for the initial cash-flow of $V(0)$ from the buyer to the writer. Of course, it would have been possible for the buyer to invest this amount in a risk-free manner, so that $V(0)$ accumulates at the common interest rate present in the market. It might be of interest to see how the payoff curves above are modified after this initial cash-flow is taken into account. In other words, we wish to look at the “future value” at time $-T$ of all of the cash-flows the two parties are involved in. In short, the realized profit for the buyer is

$$FV_{0,T}(-V(0)) + V(T)$$

and the realized profit for the writer is

$$FV_{0,T}(V(0)) - V(T).$$

Again, we ignore any intermediate cash-flows in the expressions above. But, we will reintroduce them as our derivative securities become more and more elaborate!

As a function of the argument $s$, the profit curve for the buyer has the following expression

$$FV_{0,T}(-V(0)) + v(s).$$
As a function of the argument $s$, the profit curve for the writer has the following expression

$$F_{0,T}(V(0)) - v(s).$$

3.5. **Suggested problems.** McDonald’s “Derivatives markets: #2.1, #2.2, #2.6.