University of Texas at Austin

HW Assignment 3

Provide a complete solution to the following problem(s):

**Problem 3.1.** (9 points) Assume that \( Y_1 = e^X \) where \( X \) is a standard normal random variable.

(i) (3 pts) What is the probability that \( Y_1 \) exceeds 5?

(ii) (6 pts) Find the mean and the variance of \( Y_1 \).

*Hint:* It helps if you use the expression for the moment generating function of a standard normal random variable.

**Problem 3.2.** (8 points) Source: Problem 18.6 in McDonald.

Let \( X \sim N(2, 5) \).

(i) (4 pts) Find \( E[e^X] \).

(ii) (4 pts) Find the median of \( e^X \).

**Problem 3.3.** (8 points) Two scales are used to measure the mass \( m \) of a precious stone. The first scale makes an error in measurement which we model by a normally distributed random variable with mean \( \mu_1 = 0 \) and standard deviation \( \sigma_1 = 0.04m \). The second scale is more accurate. We model its error by a normal random variable with mean \( \mu_2 = 0 \) and standard deviation \( \sigma_2 = 0.03m \).

We assume that the measurements made using the two different scales are independent.

To get our final estimate of the mass of the stone, we take the average of the two results from the two different scales.

What is the probability that the value we get is within 0.005\( m \) of the actual mass of the stone?

Provide your final answer only for the following problem(s).

**Problem 3.4.** (1 pt) Let \( Y \) be a lognormal random variable such that \( P[Y < 1] = 1/2 \). Define \( X = \ln(Y) \). What is the mean of the random variable \( X \)?

(a) \(-1\)

(b) 0

(c) 1

(d) \(3/2\)

(e) None of the above

**Problem 3.5.** (1 pt) Source: Self-test problem #5.14 from “Probability” by Ross

Suppose that the cumulative distribution function of the random variable \( X \) is given by

\[
F_X(x) = 1 - e^{-x^2/2} \quad x > 0.
\]

Evaluate \( E[X] \).

*Hint:* Use the properties of the standard normal distribution.

(a) \(\sqrt{\pi/2}\)

(b) \(\sqrt{\pi}\)

(c) \(\sqrt{2\pi}\)

(d) \(2\sqrt{\pi}\)

(e) None of the above