Turbine Example – Revised Version

The reading of the pressure drop across an expansion valve of a turbine is expected to be influenced by gas temperature on the inlet side, operator, and the pressure gauge used by the operator. A three-way design is used to study the effects of these three factors. Three temperatures are fixed. Four operators and three gauges are randomly selected. Two observations are taken at each treatment level.

A three-way complete model was used.

Minitab output, with Temp fixed, other factors random:

Analysis of Variance for Drop

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp</td>
<td>2</td>
<td>1023.36</td>
<td>511.68</td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>Operator</td>
<td>3</td>
<td>423.82</td>
<td>141.27</td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>Gauge</td>
<td>2</td>
<td>7.19</td>
<td>3.60</td>
<td></td>
<td>*</td>
</tr>
<tr>
<td>Temp*Operator</td>
<td>6</td>
<td>1211.97</td>
<td>202.00</td>
<td>14.59</td>
<td>0.000</td>
</tr>
<tr>
<td>Temp*Gauge</td>
<td>4</td>
<td>137.89</td>
<td>34.47</td>
<td>2.49</td>
<td>0.099</td>
</tr>
<tr>
<td>Operator*Gauge</td>
<td>6</td>
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<td>34.91</td>
<td>2.52</td>
<td>0.081</td>
</tr>
<tr>
<td>Temp<em>Operator</em>Gauge</td>
<td>12</td>
<td>166.11</td>
<td>13.84</td>
<td>0.65</td>
<td>0.788</td>
</tr>
<tr>
<td>Error</td>
<td>36</td>
<td>770.50</td>
<td>21.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>71</td>
<td>3950.32</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

* No exact F-test can be calculated.

Compare and contrast with treating all factors random:

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</tbody>
</table>

So far nothing different.
Displaying expected mean squares:

1. With Temp fixed:

<table>
<thead>
<tr>
<th>Source</th>
<th>Variance component</th>
<th>Error term (using unrestricted model)</th>
<th>Expected Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Temp</td>
<td></td>
<td>* (8) + 2(7) + 8(5) + 6(4) + Q[1]</td>
<td></td>
</tr>
<tr>
<td>2 Operator</td>
<td>-4.544</td>
<td>* (8) + 2(7) + 6(6) + 6(4) + 18(2)</td>
<td></td>
</tr>
<tr>
<td>3 Gauge</td>
<td>-2.164</td>
<td>* (8) + 2(7) + 6(6) + 8(5) + 24(3)</td>
<td></td>
</tr>
<tr>
<td>4 Temp*Operator</td>
<td>31.359</td>
<td>7 (8) + 2(7) + 6(4)</td>
<td></td>
</tr>
<tr>
<td>5 Temp*Gauge</td>
<td>2.579</td>
<td>7 (8) + 2(7) + 8(5)</td>
<td></td>
</tr>
<tr>
<td>6 Operator*Gauge</td>
<td>3.512</td>
<td>7 (8) + 2(7) + 6(6)</td>
<td></td>
</tr>
<tr>
<td>7 Temp<em>Operator</em>Gauge</td>
<td>-3.780</td>
<td>8 (8) + 2(7)</td>
<td></td>
</tr>
<tr>
<td>8 Error</td>
<td>21.403</td>
<td>(8)</td>
<td></td>
</tr>
</tbody>
</table>

* No exact F-test can be calculated.

2. With Temp random:

<table>
<thead>
<tr>
<th>Source</th>
<th>Variance component</th>
<th>Error term (using unrestricted model)</th>
<th>Expected Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Temp</td>
<td>12.044</td>
<td>* (8) + 2(7) + 8(5) + 6(4) + 24(1)</td>
<td></td>
</tr>
<tr>
<td>2 Operator</td>
<td>-4.544</td>
<td>* (8) + 2(7) + 6(6) + 6(4) + 18(2)</td>
<td></td>
</tr>
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<td>-2.164</td>
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<td>3.512</td>
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<td></td>
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<tr>
<td>7 Temp<em>Operator</em>Gauge</td>
<td>-3.780</td>
<td>8 (8) + 2(7)</td>
<td></td>
</tr>
<tr>
<td>8 Error</td>
<td>21.403</td>
<td>(8)</td>
<td></td>
</tr>
</tbody>
</table>

Note the differences in the row for the fixed factor Temp.

In this example, we see strong evidence of temperature by operator interaction, so it is not reasonable to test for the main effect of temperature. However, for purposes of illustration, if we had obtained no evidence for interaction of temperature with any of the other factors, and wanted to test the main effect of interaction, we would need to figure out and use the appropriate denominator.

From the expected mean squares table (1 above), we have

- \( E(\text{MST}) = (8) + 2(7) + 8(5) + 6(4) + Q[1] \)
- \( E(\text{MSTO}) = (8) + 2(7) + 6(4) \)
- \( E(\text{MSTG}) = (8) + 2(7) + 8(5) \)
- \( E(\text{MSTOG}) = (8) + 2(7) \)

If it were appropriate to test for main effect of T, then if the null hypothesis of no main effect of temperature were true, we would have \( Q[1] = 0 \), and so \( E(\text{MST}) \) would equal
\[ E(MSTO + MSTG - MSTOG) \]. Thus \( MSTO + MSTG - MSTOG \) would be the appropriate denominator. We can get Minitab to do the test as follows:

Approximate F-test with denominator: \( \text{Temp*Operator} + \text{Temp*Gauge} - \text{Temp*Operator*Gauge} \)
Denominator MS = 222.63 with 7 degrees of freedom

<table>
<thead>
<tr>
<th>Numerator</th>
<th>DF</th>
<th>MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp</td>
<td>2</td>
<td>511.7</td>
<td>2.30</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Output from a later version of Minitab:

ANOVA: Drop versus Temp, Operator, Gauge

<table>
<thead>
<tr>
<th>Factor</th>
<th>Type</th>
<th>Levels</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp</td>
<td>fixed</td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>Operator</td>
<td>random</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Gauge</td>
<td>random</td>
<td>3</td>
<td>1</td>
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Analysis of Variance for Drop

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<td>423.82</td>
<td>141.27</td>
<td>0.63</td>
<td>0.616</td>
</tr>
<tr>
<td>Gauge</td>
<td>2</td>
<td>7.19</td>
<td>3.60</td>
<td>0.06</td>
<td>0.938</td>
</tr>
<tr>
<td>Temp*Operator</td>
<td>6</td>
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<td>202.00</td>
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</table>

x Not an exact F-test.

<table>
<thead>
<tr>
<th>Source</th>
<th>Variance Error</th>
<th>Expected Mean Square for Each Component Term (using unrestricted model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Temp</td>
<td></td>
<td>*(8) + 2(7) + 8(5) + 6(4) + Q[1]</td>
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<tr>
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<tr>
<td>8 Error</td>
<td>21.403</td>
<td>*(8)</td>
</tr>
</tbody>
</table>
* Synthesized Test.

Error Terms for Synthesized Tests

<table>
<thead>
<tr>
<th>Source</th>
<th>Error DF</th>
<th>Error MS</th>
<th>Synthesis of Error MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Temp</td>
<td>6.97</td>
<td>222.63</td>
<td>(4) + (5) - (7)</td>
</tr>
<tr>
<td>2 Operator</td>
<td>7.09</td>
<td>223.06</td>
<td>(4) + (6) - (7)</td>
</tr>
<tr>
<td>3 Gauge</td>
<td>5.98</td>
<td>55.54</td>
<td>(5) + (6) - (7)</td>
</tr>
</tbody>
</table>