SMALL EXPERIMENTS – OVERVIEW

For practical reasons, it is sometimes necessary to run experiments with just one observation per cell \( r = 1 \). This creates the following problems:

- Confidence intervals may be wide and power of hypothesis tests low.
- The estimate \( \text{msE} = \text{ssE}/(n-v) \) for \( \sigma^2 \) in the Two-Way Complete model doesn’t apply, since \( n-v = abr - ab = 0 \).

Possible remedies for the second problem

1. If interaction is known in advance to be negligible, use the Main Effects model.
2. Sometimes a log or other transform can create a situation of no interaction.
3. If certain contrasts are known in advance to be negligible, and if these contrasts are “orthogonal,” then the Method of Orthogonal Contrasts may be used to pool the sums of squares of these contrasts together to obtain an estimate of error variance. (See Section 6.7.2.)
4. If the size of the interaction term is expected to be proportional to the product of the main effects \( (\alpha\beta)_{ij} = \lambda \alpha_i \beta_j \), then Tukey’s Test for Additivity can be used (See Section 6.7.3, or Montgomery Section 5-3.7.)
5. If the number of treatment combinations is large and only a few contrasts are likely to be non-negligible (“effect sparsity”) there are a couple of techniques available. (See Section 7.5.)

Cautions:

1. With no replication, we can’t expect to get much estimate of pure experimental error.
2. A method with error estimate biased \( \text{up} \) will give a conservative procedure: computed p-values will generally be larger than the true p-value. Consequence: we fail to reject the null hypothesis some times when we would reject under an “exact” method. Thus we might omit terms from models. Methods that “pool” error from interactions deemed negligible are valid or conservative, if there is no data snooping.
3. A method with error estimate biased \( \text{down} \) will give a liberal procedure: the computed p-values is generally smaller than the true p-value. Consequence: a higher rate of falsely rejecting the null hypothesis. Thus we might include unneeded terms in models. Data snooping tends to have this effect.
4. Another method sometimes used is to use an “external estimate” of error. This is risky, since we cannot be sure that an estimate obtained from a previous experiment really applies in a new experimental situation.