FACTORS AND INTERACTIONS

Example: Testing a new teaching method.

The question: Do students learn more under the experimental method than other the usual method, other things being equal?

Experimental design:

• Subjects randomly assigned to treatment and control groups.

• Post-test to measure outcomes.

• Pre-test to take into account prior learning

• Aptitude test to account for other initial differences.

Variables:

Response:

\[ y = (\text{post-test score}) - (\text{pretest score}) \]

Predictors:

\[ x_1 = \text{score on aptitude test} \quad (a \ covariate) \]

\[ x_2 = \begin{cases} 
0 & \text{control group} \\
1 & \text{experimental group} 
\end{cases} \]
Possible models:

Model I. \[ E(y \mid x_1, x_2) = \eta_0 + \eta_1 x_1 + \eta_2 x_2 \]

This says:

For the control group (\(x_2 = \_\_\_\_\_\)),
\[ E(y \mid x_1) = \]
For the treatment group (\(x_2 = \_\_\_\_\_\)),
\[ E(y \mid x_1) = \]
Possible Picture when \(\eta_0 > 0, \eta_1 > 0, \eta_2 > 0\):

Exercise: Draw pictures for other cases of coefficients (e.g., \(\eta_0 > 0, \eta_1 < 0, \eta_2 < 0\))

If Model I is correct, then
\[ \eta_2 > 0 \] says:
\[ \eta_2 = 0 \] says:
\[ \eta_2 < 0 \] says:

Is this the correct model if, for example, the new method helps low aptitude students more than high aptitude students?
Model II. Adding an interaction term $x_1x_2$ to Model I gives:

$$E(y \mid x_1, x_2) = \eta_0 + \eta_1x_1 + \eta_2x_2 + \eta_3x_1x_2$$

This says:

For the control group ($x_2 = \_\_\_$),

$$E(y \mid x_1) =$$

For the treatment group ($x_2 = \_\_\_$),

$$E(y \mid x_1) =$$

If $\eta_2 > 0$ and $\eta_3 < 0$, we have the picture:

This says: The new method does ____________ for low aptitude students than for high aptitude students.

Exercise: Draw pictures and interpret the situation when $\eta_2 < 0$ and $\eta_3 > 0$. 

or: or: