ASSIGNMENT #3: DUE FRIDAY, OCTOBER 10

I. Suppose the distribution of X given Y is uniform on (0,Y). In other words, the conditional probability density function of X given Y is

$$f(x|y) = \begin{cases} 1/y, & \text{if } 0 < x < y \\ 0, & \text{if } x < 0 \text{ or } x > y \end{cases}$$

Suppose also that Y has (marginal) probability density function

$$f_Y(y) = \begin{cases} 2y, & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

a. Find the joint probability density function f(x,y) of X and Y.
b. Find the marginal probability density function of X. Be sure to include a sketch of the region where $f(x,y) \neq 0$ in your solution.
c. Find the conditional probability density function f(y|x) of Y given X.
d. Use your answer to (c) to describe in words the conditional distribution of Y given X.

II. You may (and will probably need to) use the following facts from probability in this problem:

- If the random variable Z is a function g(X,Y) of the random variables X and Y and f(x,y) is the joint pdf of X and Y, then E(Z) = $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f(x,y)\,dx\,dy$.
- If the random variable Y is a function g(X) of the random variable X and f(x) is the pdf of X, then E(Y) = $\int_{-\infty}^{\infty} g(x) f(x)\,dx$.

a. Prove that if X and Y are independent random variables, then E(XY) = E(X)E(Y).

b. Explain how this shows that independent random variables have zero covariance (i.e., are uncorrelated).

c. Suppose that the random variable X is uniform on the interval [-1, 1]. Let $Y = X^2$. (Thus X and Y are *not* independent.) Calculate Cov(X,Y) to show that it is zero. (You will first have to figure out the pdf of X, then use this to calculate both E(X) and E(Y).) Thus the converse of the statement in (b) is not true: We have produced random variables X and Y that are uncorrelated, but not independent.

III. **M 374G**: Problems 4.2 and 4.3 (p. 79)
M 384G/CAM 384T: Problem 4.6 (pp. 79 - 80)

IV. Problem 4.7 (p. 80) *Please note*: The questions asked are about the two population mean functions in general, *not* about the specific case illustrated in Figure 4.10. Be sure to explain how you got your answers.