MORE ON DIAGNOSING DISORDERS

Consider a medical test with prevalence rate \( r \), sensitivity \( s_e \), and specificity \( s_p \). In the handout *More Probability Problems*, you expressed \( P(\text{has the disease } | \text{ tests positive}) \) in terms of \( s_e \), \( s_p \), and \( r \). Now consider \( r \) to be fixed, so this expression defines a function \( f(s_e, s_p) \) of the specificity and sensitivity. (So we are now considering \( r \) as a parameter.)

1. a. Find the limit of \( f \) as the sensitivity approaches 1 (keeping the specificity fixed). Can this limit ever be 1? If so, when?

   b. Find the limit of \( f \) as the specificity goes to 1 (keeping the sensitivity fixed). How does this compare with your answer in (a)?

   c. Recall that an ideal test would have \( P(\text{has the disease } | \text{ tests positive}) = 1 \). Based on your answers to (a) and (b), do you think it would be better to increase sensitivity or increase specificity if you want to get closer to this ideal? Does this translate to “make the test better at detecting the condition” or “make the test better at detecting when the condition is not present”?

2. An ideal test also has \( P(\text{does not have disease } | \text{ tests negative}) = 1 \). Answer questions analogous to those in problem 1 to study this question. (Please call the relevant function \( g \), so that we all use the same terminology.)

3. a. Find the rate of change of \( f \) with respect to the sensitivity.

   b. Find the rate of change of \( f \) with respect to the specificity.

   c. When is the rate of change of \( f \) with respect to the sensitivity greater than (respectively, equal to; less than) the rate of change of \( f \) with respect to the specificity? What does this say about when it is better to try to increase the sensitivity or the specificity (from the point of view of increasing \( f \))?  

4. Do an analysis analogous to problem 3 for \( P(\text{does not have disease } | \text{ tests negative}) \).

*Note:* These problems and the first problem on the handout *More Probability Problems* were based on a paper that appeared in the *Journal of Social Service Research*. The paper also pointed out that mental health assessments are often made by more informal, less thorough means than the structured diagnosis methods for which he gives prevalence rates, sensitivity, and specificity in his paper. In particular, mental health practitioners tend to look for confirmatory information to support an initial guess rather than disconfirmatory information. He also points out that misdiagnoses can lead to negative outcomes for clients. The author recommends, based on his study, that:

- Social workers should avoid mental health diagnoses if at all possible.
- Social work student should be taught about the problems with such diagnoses, and the importance of looking for disconfirmatory evidence in considering a diagnosis.
• Assessments should as much as possible focus on how severe or frequent a problem is rather than whether or not it exists.
• The profession needs to train special advocates whose focus is protecting clients from poorly made diagnoses.

What do you think about these recommendations, based on what we have studied in these problem sets?