What is probability?

Axiomatic: A function P from events to non-negative numbers satisfying:

1. $0 \leq P(E) \leq 1$
2. $P(S) = 1$ (S = certain event; sample space)
3. P(union of mutually exclusive events) = sum of P of individual events

Classical Probability ("A Priori")

- Situation: "experiment" with n equally likely outcomes
- P(A) = m/n, where A is satisfied by exactly m of the n outcomes
- E.g., toss a fair die; A = an odd number comes up
- Satisfies axioms

Pros and Cons of Classical Probability

- Conceptually simple for many situations
- Doesn't apply when outcomes are not equally likely.
- Doesn't apply when there are infinitely many outcomes

Empirical Probability ("A Posteriori" or "Frequentist"))

- $P(A) = \lim_{n \to \infty}(m/n)$, where n = number of times experiment performed, m = number of times A is satisfied.
- E.g., toss a fair die; A = an odd number comes up.
- E.g., toss a coin that is suspected of not being balances; A = heads.
- Satisfies axioms

Pros and Cons of Empirical Probability

- Covers more cases than classical
- Intuitively agrees with classical when classical applies
- Might not agree in practice with classical
- Repeating the identical experiment an infinite number of times is physically impossible
- How large must n be to give a good approximation to the limit?

Subjective Probability

- A person's measure of belief that some given event will occur.
- E.g., P(the stock market will go up tomorrow).
- Satisfies axioms, if certain "coherence conditions" are followed. (e.g., P(stock market goes up tomorrow) = .6 and P(stock market goes down tomorrow) = .7 are inconsistent.)

Pros and Cons of Subjective Probability

- Applicable in situations where other definitions are not.
- Fits intuitive sense of probability.
- Can be considered to extend classical.
- Can vary from individual to individual
- Requires "coherence" conditions; are people always that rational?

Empirical(Frequentist) vs Subjective Probability in Statistics

- Classical statistics (confidence intervals, hypothesis tests) uses empirical probability.
- Classical statistics concepts often misinterpreted as if probability were subjective
- Bayesian statistics can model subjective probability.